Structural Analysis of Torsion Type Gates

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Information is given to show various analytical methods which are based upon simple torsion theory, bending- torsion theory or simplified ideas to facilitate quick calculation. The relationship between each method is clarified to help choose a proper one for various analytical purposes. The impact of bending- torsion appears more strongly in stress distribution of gate sections rather than internal forces of the section and can be calculated by the finite element method. Sound designs would not be obtained for super large gates of torsion type without considering gate deformation due to bending- torsion as well as simple torsion. Although torsion type structures have a several essential advantage of structural nature, their application is not so common. A difficulty of structural analysis is one of the reasons.

Key words: gate, torsion, bending- torsion, structural analysis, closed thin shell

1. Introduction

External forces acting on structures are grouped into loads and their reaction forces, and the loads are transmitted from loading points to reaction points through rigidity of the structure. The structural rigidity sometimes refers to a shearing rigidity, a bending rigidity, a torsion rigidity, an axial rigidity and so on, but a structure generally provides all these rigidities together and a major rigidity among them depends upon not only their relative magnitudes but also distribution of external forces. A structure most of whose external forces compose couples and whose torsion rigidity is comparatively large, is characterized by the torsion rigidity and is defined as a torsion type structure in this paper and gates of torsion type structure are defined as torsion type gates.

A flap gate of the torsion type structure is called a fish belly flap in Europe and a fabrication record shows this type was in operation at 1931 already.[6)] The first application of this type in Japan was in 1963 at Matsukawa diversion works whose outline is shown in Fig.- 1 [1) & 2)]. Since then this gate type had quickly spread all over Japan and then the next stage of applications such as multiple step fish- ladder gates or double gates as shown in Fig- 2 appeared. Furthermore, torsion type structure was applied for shipyard repair dock gates as shown in Fig.- 3 and 4 [4)] , one of which is 100m in width and more than one thousand tons of steel in weight. A superiority of torsion type structure is (1) small steel weight when it is applied to a low height long span gate and (2) advantage in fatigue strength because of almost pure shearing status of stress distribution, and this type have been proposed for gate plans in the study of alternatives for the Panama canal [5)]. The result of this study suggests that torsion type gates could

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become larger in scale and replace roller gates, miter gates or other gate types. Fig.- 5 shows a laterally moving torsion type gate of 27.5m height, 200m wide and weighing approximately 6000 tons. The gate consists of left and right blocks which are structurally

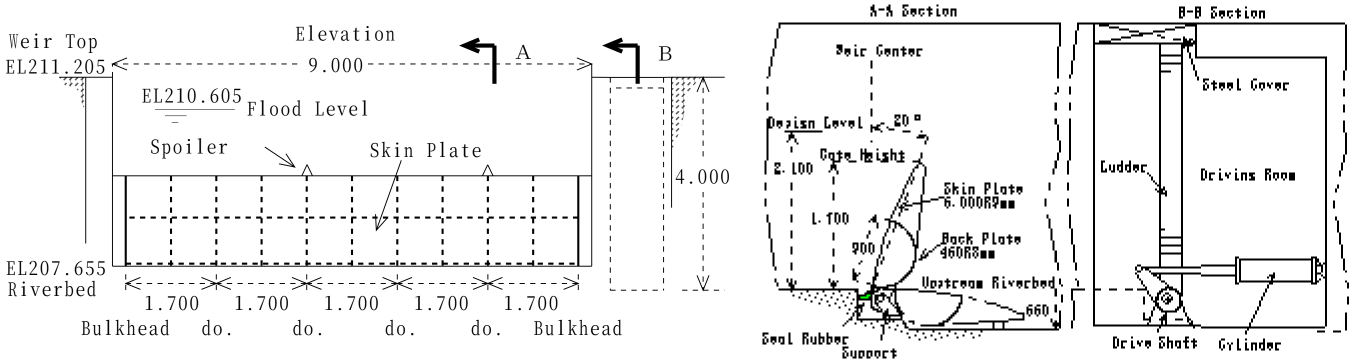


Fig.- 1 Matsukawa Diversion Work

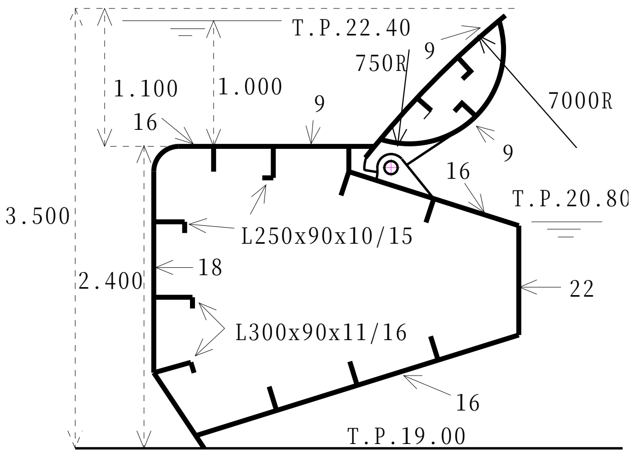


Fig.- 2 Double Gate 3.5m x 40m

13.5 m x 100 m

Fig.- 3 Dock Gate

12 m x 80 m

Fig.- 4 Dock Gate

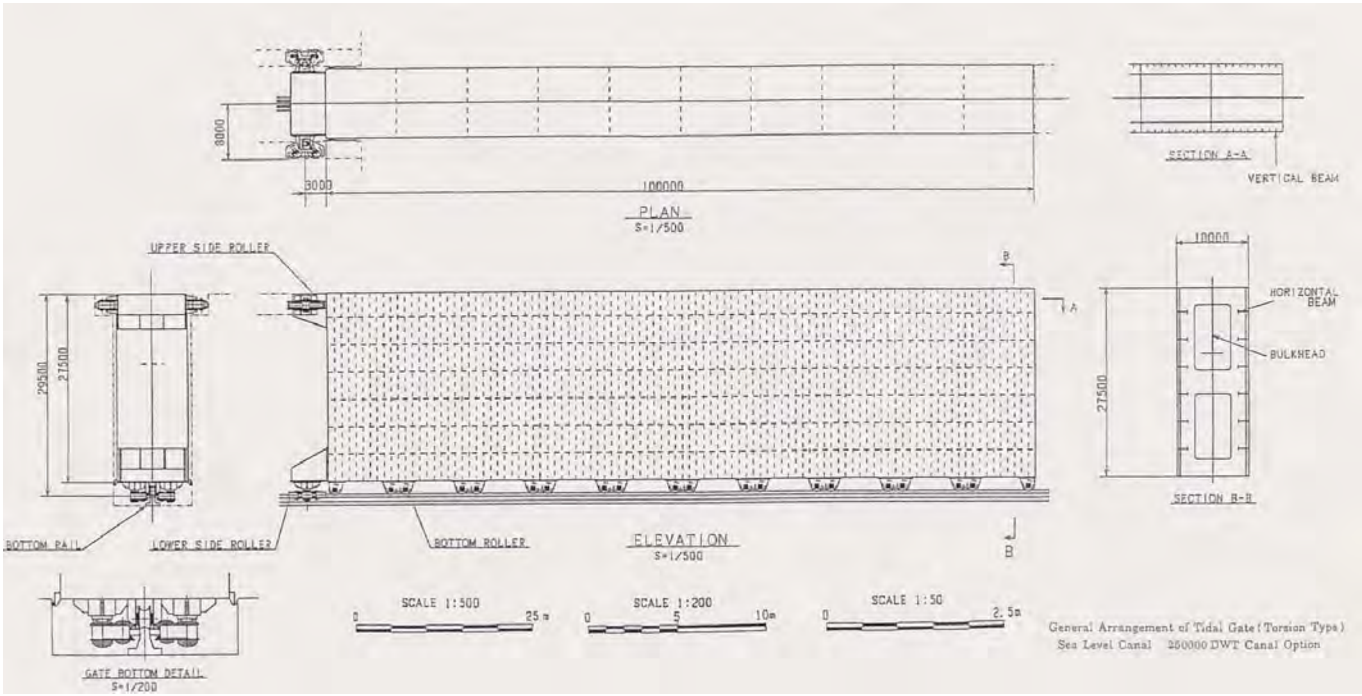


Fig.- 5 Laterally Moving Gate 27.5 m x 200 m x 5950 ton (Figure shows the left half only) independent of each other. The figure shows the concept only and the details shown are tentative [5).

Although torsion type structure in Europe seems to be applied mainly to flap gates, their development in Japan is in a somewhat different direction. Gate shapes in Japan sometimes differ from fish belly shape because much effort was made to increase torsion rigidity while to decrease bending rigidity of the gate sections and it is often inappropriate to call them "fish belly type" . This is the reason why the expression "torsion type structure" or "torsion type gate" was introduced in this paper.

The purpose of this paper is to clarify the contents of analytical methods used in the past and to define their relations. The essential part of the analysis is to obtain many statically indeterminate values and this process is extremely complicated. Although torsion in a structure is a combination of simple torsion and bending- torsion, the latter is not considered in usual structural analysis. But torsion type structures cannot be described without bending- torsion. Especially sound designs would not be obtained for super large scale gates of torsion type without considering stress distribution due to bending- torsion as well as simple torsion. The paper gives detailed explanation of analytical methods based upon the simple torsion theory, and then shows how the result will change due to the bending- torsion theory. Finally there is a brief discussion on simplification of the analysis.

2. Analysis based upon the simple torsion theory

2. 1 Stress distribution on gate sections

At the starting point of the description of the simple torsion theory, specific features in stress distribution on a closed thin shell are given shortly. Various kinds of stress are created on the vertical section of the shell. The stress includes shearing stress τ_s due to simple torsion, bending stress σ_{bx} and σ_{by} due to bending moment m_y and m_x around the y- axis and x- axis, and, shearing stress τ_{bx} and τ_{by} due to shearing force Q_x and Q_y in the x- direction and y- direction. Distribution of shearing stresses are governed by shear flow on the section. Shear flow for τ_s is constant throughout the section. Shear flows for τ_b are shown on Fig.- 8 and 9 which give the results of calculation on sections shown in Fig.- 6 and 7. G, S, to and ti in Fig.- 6 are center of gravity, shearing center and thickness of the shell section respectively. The x- axis on Fig.- 6 is set vertically for the sake of convenience in a comparison of analytical results to a reference later. Nevertheless, the x- axis in Fig.- 8 is horizontal because the figure shows the sectional view from reverse side of the paper and is rotated by 90 degrees clockwise. Sectional shapes and dimensions are also shown in the figures. The shear flow plotted outside of the section has a plus value which corresponds to the shear flow in the clock- wise direction and the value was multiplied by rates shown in the figures for convenience of graphical display.

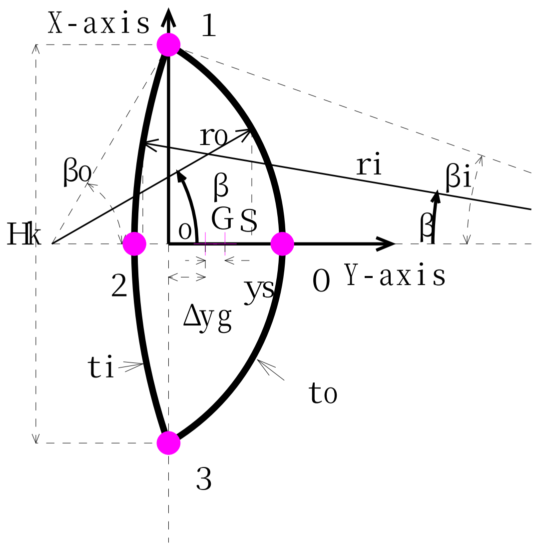


Fig.- 6 Fish Belly Shape

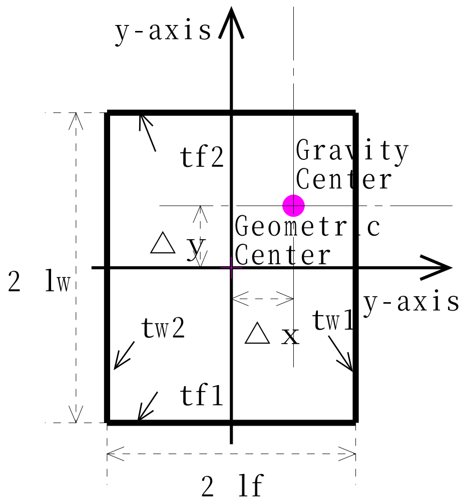


Fig.- 7 Rectangular Shape

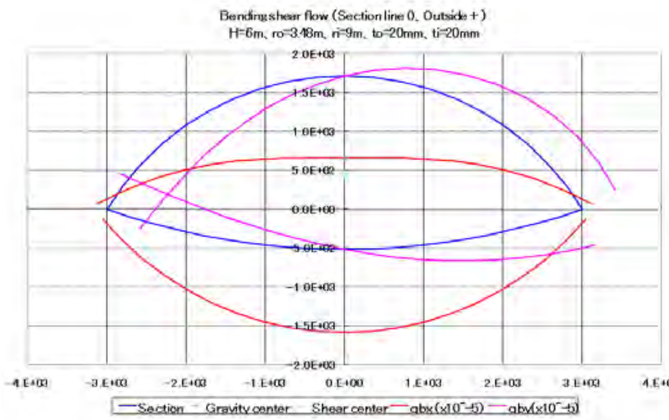


Fig.- 8 Fish Belly Shape

Fig.- 9 Rectangular Shape

2．2 Analysis by elastic equation

This is a fundamental analytical method of torsion type structure. Elastic displacement at supports of a gate is given in the form of elastic equations and internal forces are determined so that all support conditions may be satisfied. The main flow in the analysis follows the paper [7]. The equations generated are erected to a matrix equation whose solutions are obtained by a computer.

a) Analytical model

The gate to be analyzed is replaced by a shear centerline of the gate and it is assumed that bending rigidity is also concentrated along this line. Bending rigidity in the case of simple beam theory is supposed to be concentrated along the gravity center line, but no difference would occur even if the line moves to the shear center line. Load for bending deformation has to be located on the shear center line.

b) External loads in analysis

Distributed load on the gate body is replaced by concentrated loads on each web plate. The magnitude of concentrated loads is equal except at both ends where the loads are half of the others. These loads are reacted at bottom supports of the web plates and at drive ends of the gate body. Reaction forces at the gate end creates a reaction moment M_o by which the gate end is supported. Let us consider one end drive because both ends drive can be deemed to be special cases of one end drive. Reaction forces at bottom supports are divided into (W_x', W_y') which are equal to loads (W_x, W_y) on the web plates in magnitude and (X, Y) which are remainder portions of reaction forces and their + directions are defined as in Fig.- 10. We call the former statically determinate values and the latter statically indeterminate values. (W_x, W_y) and (W_x', W_y') compose couples, which we call statically determinate torsion moments represented by m_s for all sections

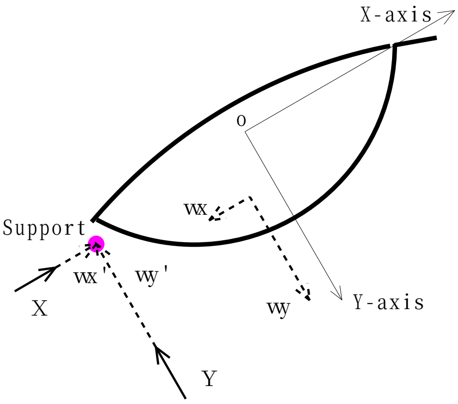


Fig.- 10 Direction of Forces

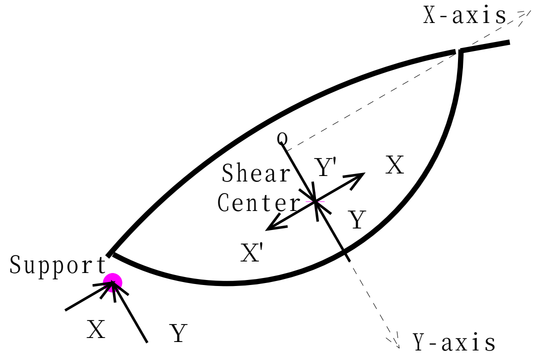


Fig.- 11 Assumed Forces

except at both ends where their value is half. We now think about the remainder (X, Y). As indicated in Fig.- 11, let us imagine that (X, Y) and (X', Y') which are equal to (X, Y) in magnitude but have opposite directions act at shear centers on the gate sections. The remainder (X, Y) and imagined (X', Y') create couples which we call imagined statically indeterminate torsion moments or m_{fi} where i denotes sectional number. The imagined (X, Y) acting on the shear centers cause bending deformation in the gate body.

c) Deformation formulas

Torsion and bending deformation will occur in the gate body due to external loads mentioned in the preceding article. As each section is supposed to have structural members different from other sections, the shear center line would be a curve and bending deformation due to imagined (X, Y) would be followed by internal torsion deformation whose estimation procedure is necessary besides for torsion and bending deformation due to external load.

1) Deformation due to concentrated torsion moment

Fig.- 12 shows boundary conditions to be considered. End a is fixed for rotation and end b is free for rotation and displacement. 0 ~ n denotes support numbers and l_s x n equals the gate length. The line which represents the gate is a curve which consists of straight lines parallel to z- axis between bottom supports. Concentrated torsion moment m_i

acting on i section creates internal torsion moment shown in Fig.- 12. The i section rotates according to the internal moment and the rotation angle on the section will be a sum of the torsion angles along the gate body between support 0 and i. The shear center on the section moves according to the sectional rotation but <u>this movement will not be counted for the reason that sectional rotation is always very small</u>. If this premise does not hold we would have to handle equations including non- linear load terms.

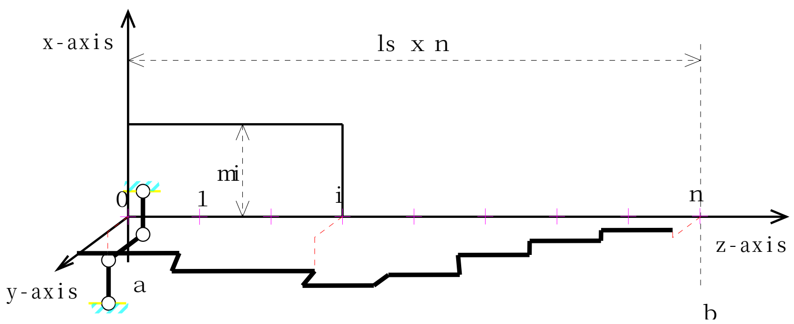


Fig.- 12 Boundary Condition

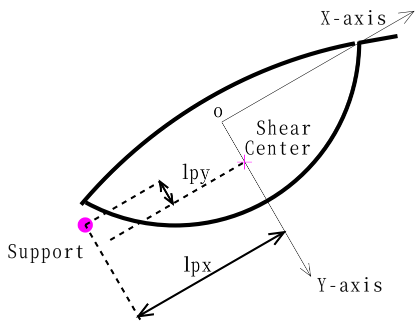


Fig.- 13 Bottom Support

Fig.- 13 shows the distance between shear center and bottom support, l_{px} and l_{py}. ξ_{tij} and η_{tij} denote the displacement of bottom support in x and y directions at j section due to m_i and they are given by formula (1) where J_t denotes torsion section modulus. For a uniform structure, formula (2) is applicable.

[Deformation at support j due to m_i]

$$\xi_{tij}=\sum_{k=1}^{j}\frac{m_i l_s}{GJ_{tk}}l_{pyk} \quad \text{for } J>i \quad \sum_{k=1}^{i}\frac{m_i l_s}{GJ_{tk}}l_{pyk}, \qquad \eta_{tij}=-\sum_{k=1}^{j}\frac{m_i l_s}{GJ_{tk}}l_{pxk} \quad \text{for } J>i \quad -\sum_{k=1}^{i}\frac{m_i l_s}{GJ_{tk}}l_{pxk} \qquad \cdots\cdots(1)$$

[Deformation at support j due to m_i:for uniform section]

$$\xi_{tij}=\frac{m_i l_s j}{GJ_t}l_{py} \quad \text{for } J>i \quad \frac{m_i l_s i}{GJ_t}l_{py}, \qquad \eta_{tij}=-\frac{m_i l_s j}{GJ_t}l_{px} \quad \text{for } J>i \quad -\frac{m_i l_s i}{GJ_t}l_{px} \qquad \cdots\cdots(2)$$

2) Bending deformation due to concentrated load

Fig.- 14 shows boundary conditions to be considered. Displacement at end a on the actual gate is restrained but end b is movable keeping equilibrium between reaction forces and loads, and the magnitude of the movement is given as solutions of the elastic equations. W_i in the figure denotes concentrated load acting on i section. ξ_{wij} and η_{wij} denote displacement in x and y directions respectively of j section due to W_i and given

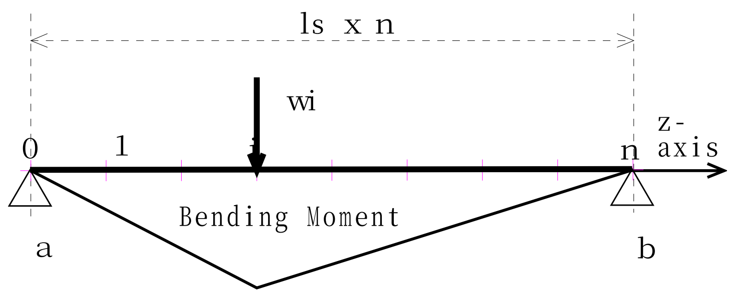


Fig.- 14 Boundary Conditions

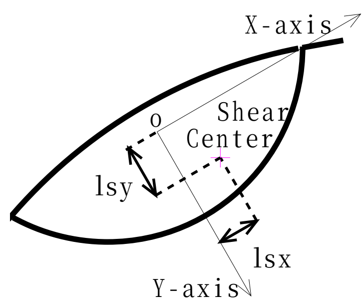


Fig.- 15 Shear Center

by formula (3) where I_x and I_y denote geometrical moment of inertia around x and y axis which coincide with the major axes of the gate section.

[Displacement at j support due to w_i on i section]

$$\xi_{wij}=X_i\sum_{k=1}^{n}\frac{l_s^3}{EI_{yk}}A_{ijk},\quad \eta_{wij}=-Y_i\sum_{k=1}^{n}\frac{l_s^3}{EI_{xk}}A_{ijk} \quad\cdots\cdots(3)$$

where, $A_{ijk}=(n-j)(n-i)\{k^3-(k-1)^3\}\div(3n^2)$ $(k\leqq i, k\leqq j)$, $B=\{3k^2-3n(k-1)^2-2k^3+2(k-1)^3\}$

$(n-i)jB\div(6n^2)$ $(j<k\leqq i)$

$(n-j)iB\div(6n^2)$ $(i<kk\leqq j)$

$ji\{(n-k+1)^3-(n-k)^3\}\div(3n^2)$ $(k>i, k>j)$

3) Deformation due to internal torsion

3-a) Internal torsion moment

The boundary condition shown in Fig.-12 is applicable. Fig.-15 shows l_{sx} and l_{sy} which denote distances of shear center from z-axis in x and y direction on the section. The internal torsion moment is given by the following formulas which are obtained through equilibrium condition of moments around the z-axis.

[Reaction forces at 0 and n section due to load at i section]

$$R_{x0i}=X_i\cdot(n-i)\div n,\ R_{y0i}=Y_i\cdot(n-i)\div n,\ R_{xni}=X_i\cdot i\div n,\ R_{yni}=Y_i\cdot i\div n \quad\cdots\cdots(4)$$

[Reaction moment at 0 section due to load at i section]

$$m_{xi}=X_i l_{syi}-R_{x0i}l_{sy0}-R_{xni}l_{syn},\ m_{yi}=Y_i l_{sxi}-R_{y0i}l_{sx0}-R_{yni}l_{sxn} \quad\cdots\cdots(5)$$

[Torsion moment between j-1 and j section due to load at i section]

$$m_{xij}=X_i l_{syi}-R_{x0i}l_{syj}-R_{xni}l_{syn}\ (j\leqq i),\ m_{yij}=Y_i l_{sxi}-R_{y0i}l_{sxj}-R_{yni}l_{sxn}\ (j\leqq i) \quad\cdots\cdots(6)$$

$$=R_{xni}(l_{syj}-l_{syn})\ (j>i) \qquad =R_{yni}(l_{sxj}-l_{sxn})\ (j>i)$$

3-b) Torsion deformation

Deformation due to m_{xij} and m_{yij} will be given by formula (1) after m_i in the formula is replaced by them. (ξ_X, ξ_Y) and (η_X, η_Y) denote displacement at bottom supports in x and y direction due to imaginary (X, Y) and are given by following formulas.

[Displacement at j support due to (X_i, Y_i) on i section]

$$\xi_{Xij}=\sum_{k=1}^{j}\frac{m_{xik}l_s}{GJ_{tk}}l_{pyk},\ \eta_{Xij}=-\sum_{k=1}^{j}\frac{m_{xik}l_s}{GJ_{tk}}l_{pxk},\ \xi_{Yij}=\sum_{k=1}^{j}\frac{m_{yik}l_s}{GJ_{tk}}l_{pyk},\ \eta_{Yij}=-\sum_{k=1}^{j}\frac{m_{yik}l_s}{GJ_{tk}}l_{pxk} \quad\cdots\cdots(7)$$

d) Elastic equation

Elastic equations for support points are erected to a matrix formula as follows. F is a

$$F\chi=S \quad\cdots\cdots(8)$$

square matrix having 2n+2 lines, χ is a column vector consisting of 2n+2 unknowns and

$$\chi=F^{-1}S \quad\cdots\cdots(9)$$

S is a column vector having 2n+2 constant elements. F is a coefficient for χ. Fig.-16 shows detailed contents of each matrix. Solutions of formula (8) are given by formula (9) where F^{-1} is an inverted matrix of F. Matrix operation required to obtain χ can be carried out quite easily with the assistance of a computer. For both end drives, the theory described in this section can be applied in its entirety except that half of statically determinate torsion moment M_0 is applied at n section. The difference will exist only in column vector S of formula (9) and (X, Y) of both end drives would agree with the results of one end drive if the gate section is uniform throughout.

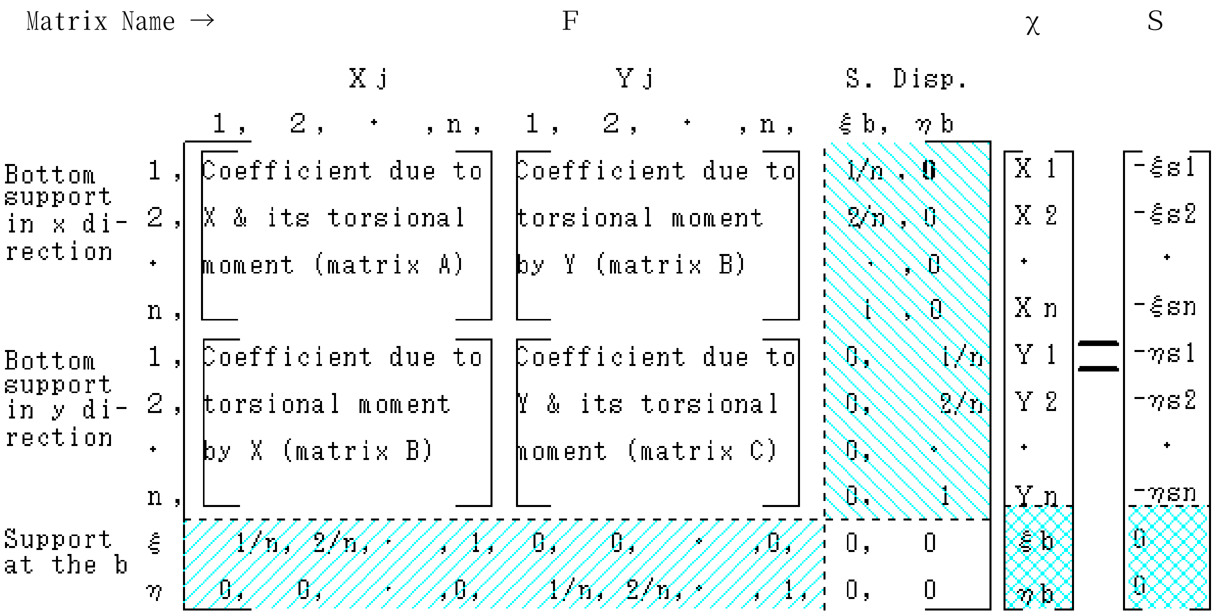


Fig.- 16 Elaments of Matrix

e) Results

Stress distribution is obtained from the internal sectional forces which are obtained from the statically indeterminate reaction forces given as solutions of the matrix formula. Three examples are shown below.

[Example 1]

This is an ordinary case whose gate body has a uniform section and is driven at one end. This case will be compared with the results of bending- torsion theory later. Conditions for the calculation are as follows. Gate height: H_g= 6400 mm, Gate width: L_g = 25000 mm, Inclination: θ=15°, Bottom support: L_{py} = 391 mm, L_{px} = 3206 mm, Number of support spaces: 8, Calculation angle: 30°, Section: H_k = 6000 mm, r_o = 3480 mm, r_i = 9000 mm, t_o = 20 mm, t_i = 20 mm, Symbols other than specified in Fig.- 17 follow Fig.- 6. Fig.- 8 corresponds to this example. Figs.- 18 ～ 20 show results of the analysis where X and Y axis correspond to those defined in Fig.- 17 and symbols are common to those appearing at other places in this section (2). Fig.- 18 shows support reaction force, shearing force and bending moment and Fig- 19 shows torsion moment. The lateral axes in these figures represent section numbers and the vertical axes represent analytical results after multiplyed by rates shown in the figures. As the statically indeterminate reaction force of a sectionally homogeneous case is very small, the corresponding shearing force and bending moment are also very small and, eventually, torsion moment almost equals to the statically determinate torsion moment and decreases almost lineally toward the unsupported gate end. Fig.- 20 shows shearing stress, normal stress and principal stress on section 1. The lateral axis represents locations on the section and numbers on the axis agree with those in Fig.- 6.

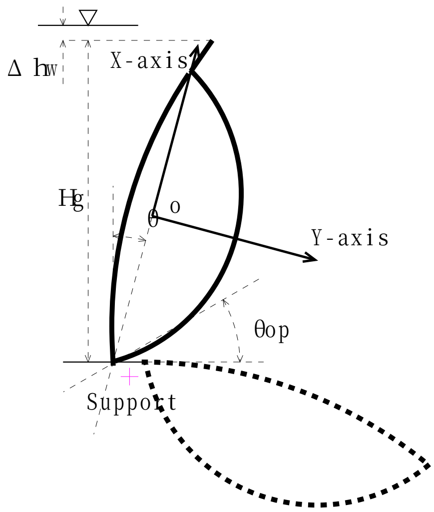


Fig.- 17 Symbols and coordinates

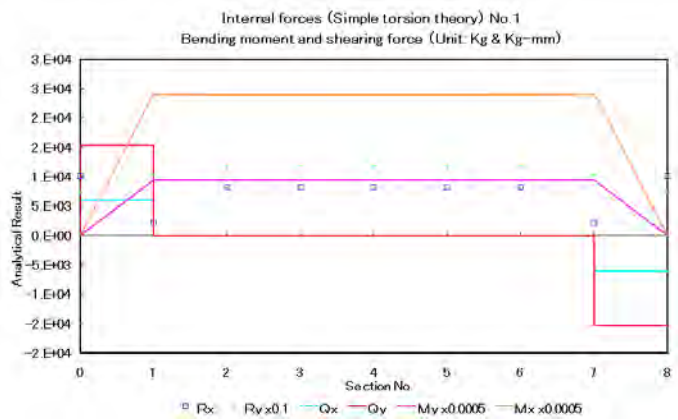


Fig.- 18 Bending Moment etc.

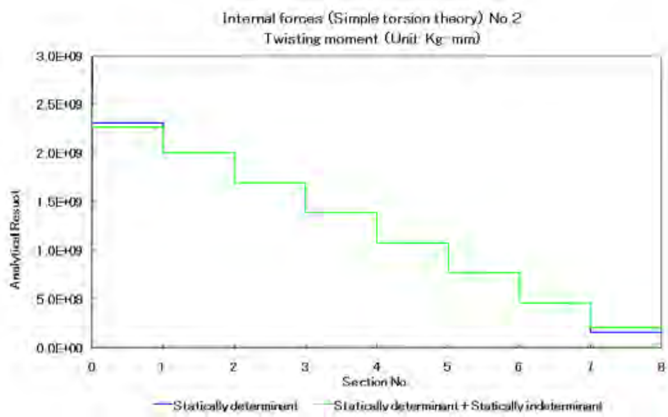


Fig.- 19 torsion Moment

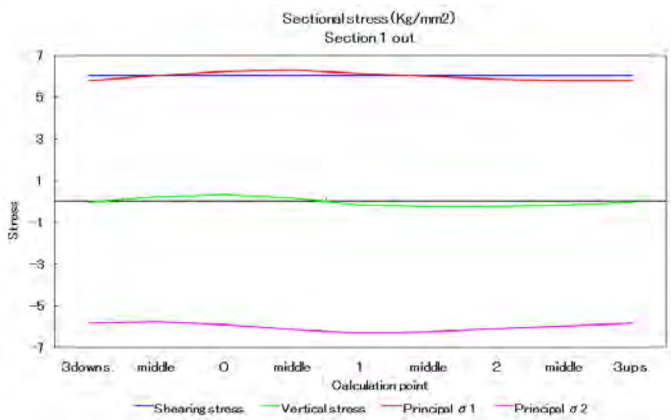


Fig.- 20 Stress

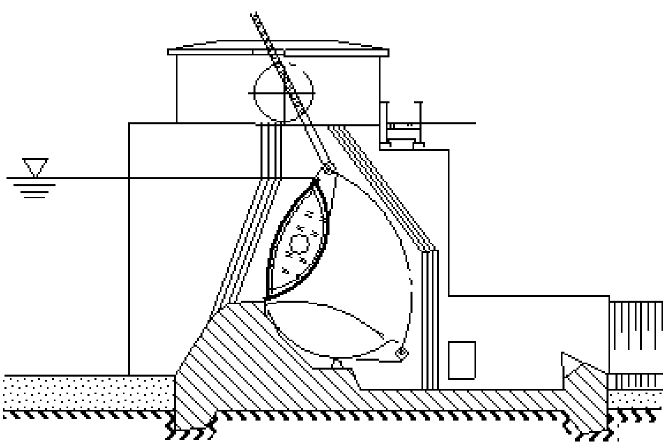


Fig.- 21 Gate Analyzed

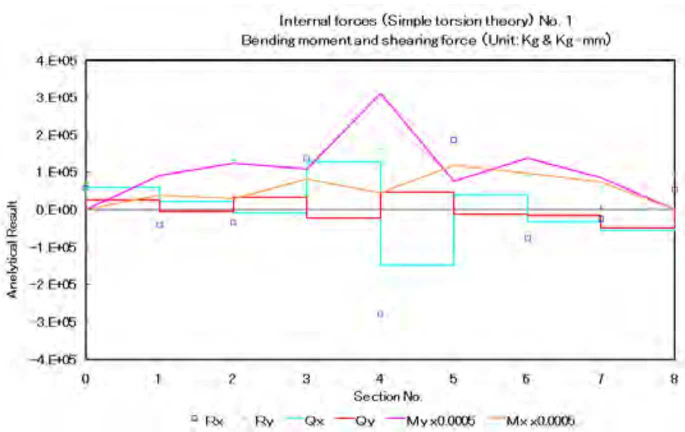


Fig.- 22 Bending Moment etc.

[Example 2]

This case intends to show the difference in internal forces of sectionally non homogeneous structure which was taken from reference [7]. Fig.- 21 shows an analyzed gate whose width is 50 m with both ends supported but structurally separated at its center. Eventually each half of the gate is independent and its length is 25 m which is divided into 8 spaces by 9 bottom supports. Sectional rigidity at section 0～2 is maximum, then section 2～4, and section 4～8 is minimum. Example 1 conforms to this case as far as the gate length and number of bottom supports are concerned, and the both sectional shapes are almost same. Fig.- 22 shows bottom support reaction, shearing force and bending moment. They exactly correspond to the original figures except that their + directions agree with the definitions made in section (2). Note that the scale on the vertical axis and the figures to be multiplied to the analytical results do not agree with Fig.- 18. Sectionally non homogeneous structure result in a wide increase of sectional forces and disturbance in their distribution. All of them show the impact of a remarkable increase in statically indeterminate reaction forces that agrees with the results obtained by the author in his own projects. Nevertheless it is confirmed that there is no big variation in internal torsion moment and the overall tendency is similar to Fig.- 19. The reference cited also shows that m_y decreases remarkably when the restraint in x- direction on the bottom supports is released, because of the reduction of slope change rate in gate deformation.

[Example 3]

This case is of a sectionally homogeneous rectangular gate driven at one end and will be compared with bending- torsion theory later. Fig.- 23 shows symbols and a set of coordinates with the gate section at installed position. X- axis passes through shear center and a bottom support. Bending moment around y- axis would not be created in this arrangement even if the section rotates around the support. Conditions for analysis are as follows. Gate height: H_g = 27500 mm, Gate width: L_g = 100000 mm, Inclination: θ = 0 °, Bottom support: L_{py} = 0 mm, L_{px} = 15750 mm, Number of support spaces: 8, Calculation angle: 0 °, Section: L_f = 5000 mm, L_w = 13750 mm, t_{f1} = 34 mm, t_{w1} = 34 mm, t_{f2} = 34 m, t_{w2} = 34 mm. Symbol other than defined in Fig.- 23 follows Fig.- 6 and Fig.- 7.

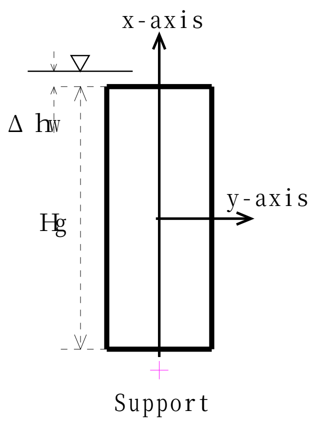


Fig.- 23 Symbol and Coordinates

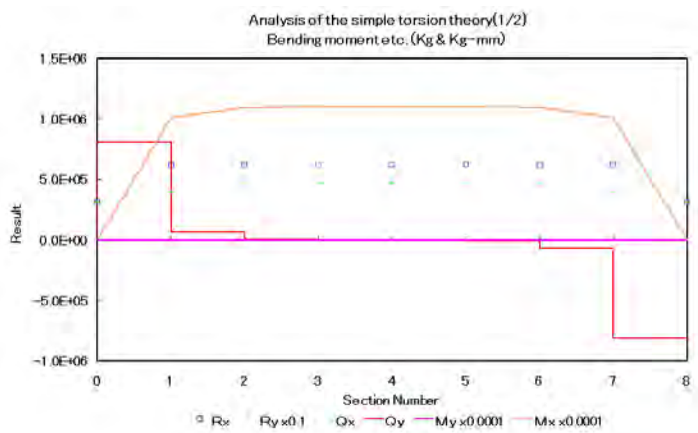


Fig.- 24 Bending Moment and shearing force

Fig.- 9 corresponds to this case. Fig.- 24 shows bottom reaction, shearing force and bending moment. Indication of internal forces accords with the coordinates defined in

Fig.- 17 and + direction corresponds to the definitions made for the calculation formulas in this section.

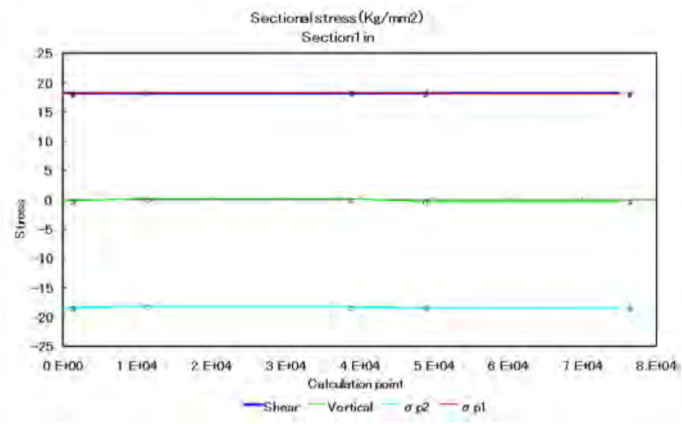


Fig.- 25 Stress

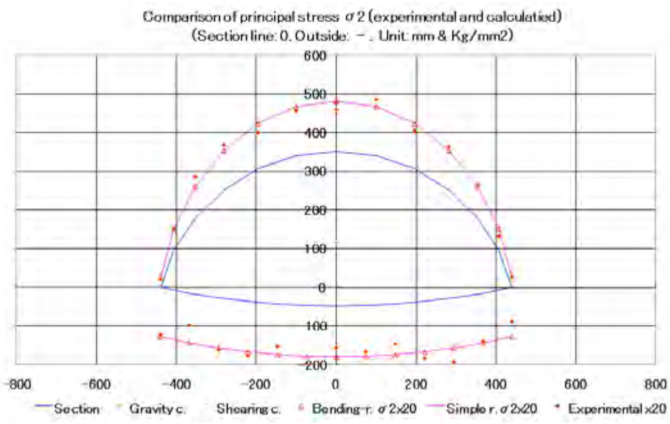


Fig.- 26 Principal Stress

Fig.- 25 shows distribution of shearing stress, normal stress and principal stress on inside (= gate center side) of section 1. Indication of stress accords with the coordinates defined in Fig.- 7. The lateral axis corresponds to Gauss length along the section and letters shown agree with those in Fig.- 7.

f) Model test

Large scale model experiment (17m width x 1m height) was carried out to verify the validity of analytical result of the elastic equations[1)&2)]. Stress distribution and deformation analyzed showed good agreement with experimental results in general. Fig.- 26 is an example from the result and shows principal stress distribution on the model section which is located at the intermediate of section 0 and 1. Simple σ_2 on the figure denotes analytical result. Stresses were multiplied by 20 for convenience of graphic display.

g) Effect of bottom support move

As an example of application of the elastic equation, analysis of the operation load of a flap gate on main gate shown in Fig.- 2 when the main gate deflects due to water load is shown. Formulas (10)～(12) give M_d which is the increase in operation load when the flap

$$M_d = M_f + \frac{dU}{d\theta} \qquad \cdots\cdots(10)$$

$$\frac{dU}{d\theta} = \delta\,(u_x - u_y)\sin 2(\theta_d + \theta) \qquad \cdots\cdots(11)$$

$$M_f = r_p c_{pf}\,\delta\,\{W_{xp}\cos(\theta_d + \theta) + W_{yp}\sin(\theta_d + \theta)\} \qquad \cdots\cdots(12)$$

is rotated through angle θ in the clockwise direction where U denotes strain energy accumulated in the gate body due to operation, δ denotes displacement of a key support whose movement represents all other bottom support movement, and, u_x and u_y denote strain energy created in the gate body when the key support moves one unit length in the x and y direction respectively, θ_d denotes angle of δ from x- axis measured in anti- clockwise direction, M_f denotes frictional torque on a support pin, r_p denotes radius of the pin, c_{pf} denotes frictional coefficient of a pin bearing, and, w_{xp} and w_{yp} are frictional forces on the support pins when the key support moves one unit length in x or y direction respectively. U_x and U_y can be obtained through torsion and bending moment diagrams which are calculated from resolutions of elastic equations for unit support displacement, and, w_{xp} and w_{yp} are given by following formulas where X_k and Y_k are statically indeterminate reaction forces at section k. X and y axes are supposed to coincide with principal axes of a gate section. Increase in operation load was

$$W_P=(W_{PX}{}^2+W_{PY}{}^2)^{1/2} \quad \text{where,} \quad W_{PX}=\sum_{k=0}^{n}|X_k| \text{ 、 } W_{PY}=\sum_{k=0}^{n}|Y_k| \qquad \cdots\cdots(13)$$

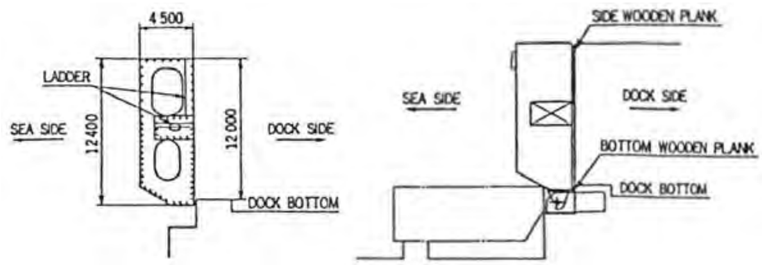
calculated on a gate shown in Fig.- 2 and it was found that dU/ d θ is about 1% of the moment due to gate weight and about 0.5% of the total moment including water pressure and m_f is about 0.5% of the moment due to seal rubber friction. Although these increase are practically negligible, the increase in support reactions is about 10% which has to be taken into account for gate leaf design. Increase in operational load was measured on the model referred to in article f) and no meaningful difference was found between the values before and after bottom supports moved.

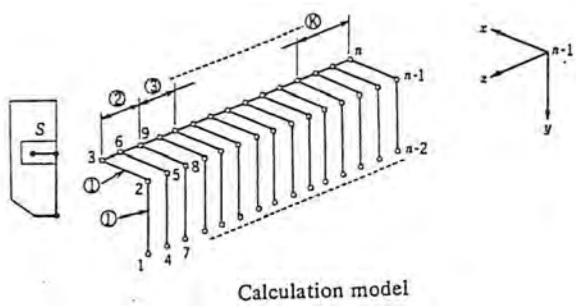
2. 3 Analysis by space frame theory

It can be shown that the dock gate on Fig.- 4 is replaced by a space frame whose analytical result are obtained through computer software in general use.

a) Analytical model

Fig.- 27 shows the gate section to be analyzed. This gate is a kind of torsion type gate with a closed thin shell having web frames arranged at a constant interval and a heavy terminal wall which can resists great terminal reaction force, but there are no supports along the gate bottom except a vertical dock terminal concrete sill wall against which the gate bottom with its wooden bearing seats, is pressed by water pressure on the gate. Fig.- 28 shows the analytical model which consists of the closed thin shell replaced by 3 ～6, 6 ～9 etc. passing through sectional shear center and the web frame members replaced by 1 ～2, 2 ～3 etc. An analytical model of space frame structure is usually built on the assumption that rigidities in each structural member are concentrated along at the sectional center of gravity because the bending moments and axial forces play main roles in the structural deformation. Even in the case where torsion rigidity is not negligible, problems seldom arise because the sectional shear center of members in most space frames coincides with the sectional center of gravity. In the case of torsion type structures, both centers usually do not coincide and if it is assumed that bending rigidity of a torsion member is concentrated along its shear center, the joint interval of the member next to the torsion member would change. The interval change is bound to bring about the same result as rigidity change of the next member, and the rigidity has to be adjusted according to the interval change so that proper internal forces may be obtained through computer analysis. Rigidities of the web frame members in Fig.- 28 are supposed to be adjusted to take into account the disagreement between the gate body sheer and gravity centers but this model could be justified by giving so larger rigidity to the web members that their deformations are deemed to be zero, that is a tacitly accepted premise in the analysis by elastic equations.





Middle Terminal

Fig.- 27 Section

Fig.- 28 Analytical Model

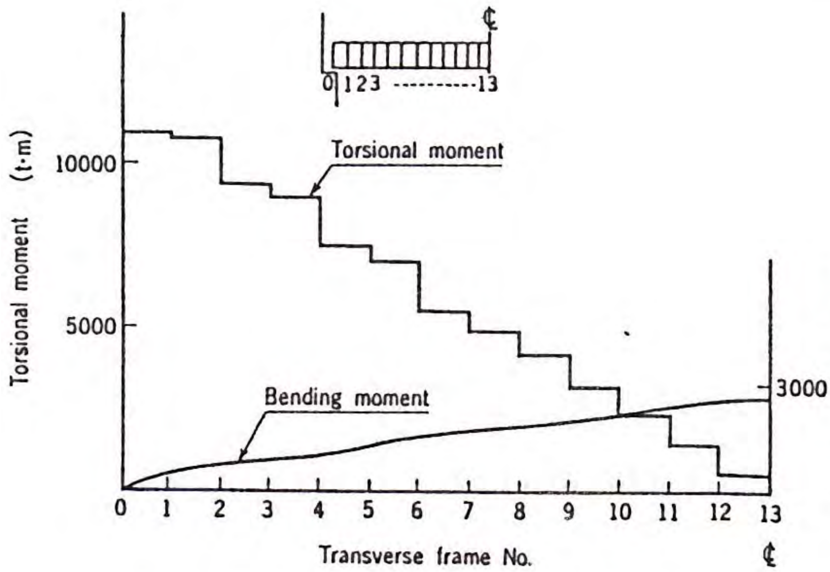


Fig.- 29 Results

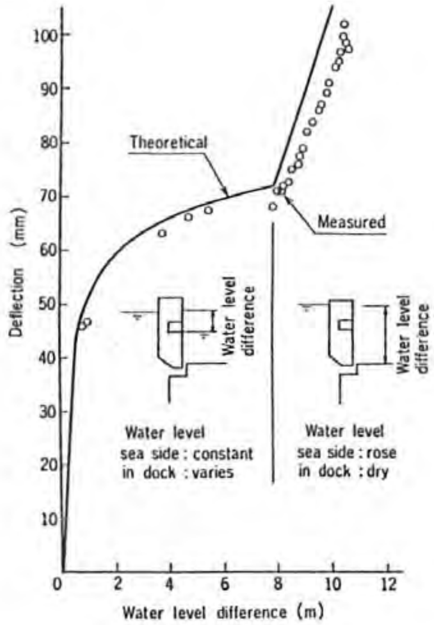


Fig.- 30 Deflection

b) Analytical results

Fig.- 29 shows torsion moment and bending moment in the gate body. Fig.- 30 shows analytical and experimental result of gate top displacement at test loading. Both results show fairly good agreement. Similar agreement was also obtained for stress values at the gate top. Analytical result of space frame theory is supposed to be same as those obtained from analysis by elastic equations.

c) Lateral strength

Although analysis by space frame theory was shown with the assumption that no deformation arises on vertical gate sections including web frames, actual gate section deforms to some extent and separate analysis is necessary to clarify the effect of these deformation on the gate strength. Type of deformation at terminal, intermediate and center sections are different and a proper analytical method for each section has to be chosen. As one example, the method to analyze a terminal section including a heavy terminal wall is shown. External load elements on the section are bending and torsion shearing stresses working through the closed thin shell section, a terminal load which is transmitted through stiffeners on shell plate of the water sealing side, M_o (reaction moment) which is described at b) of section 2. (2), and, statically determinate and indeterminate reaction forces (w and X) acting on the terminal support. These elements are in equilibrium within each combination shown on table 1. Fig.- 31 shows working points of terminal reaction force R_H and wooden bearing seat reaction R_T which form a

Number	Conbination Elements
①	Statically indeterminate reaction force & Integrated Bending shearing stress
②	Statically determinate reaction force & Terminal load
③	Reaction moment & Coupling of Torsional shearing stress ,Combination ① and Combination ②

Table- 1 Load Combination on Terminal Section

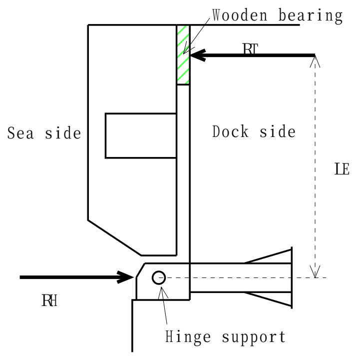


Fig.- 31 Terminal Reactions

couple of M_o. L_E is arm length of the couple. Since magnitude of M_o is defined by statically determinate torsion moment, R_T and R_H are obtained through the following formulas. Subscripts x denotes x- direction and the figure 1 denotes joint number 1. R_T

$$R_T = M_0 \quad L_E \qquad \cdots\cdots(14)$$

$$R_H = R_T - (w_{x1} + X_1) \qquad \cdots\cdots(15)$$

is supposed to be distributed approximately uniformly along the wooden bearing seat. RH is total reaction force of the terminal support in x direction and the reaction force in y direction is given approximately by the gate weight multiplied by (3/ 16). The terminal load is shared by the stiffeners. Internal force in the section can be analyzed by two dimensional finite element method or by manual calculation after the section is replaced by a statically determinate structure. Total load on the terminal support is remarkably larger than reaction forces at intermediate support points of the analytical model because of the existence of R_T.

d) Shear buckling

Precautions against shear buckling is very important for a torsion type structure because stress in shell members of closed sections is almost purely shear. Bucking strength for shearing force can be verified by formulas listed in any handbook for structural engineering. Openings are provided on shell members for various purposes. Consideration has to be given to the decrease in shear buckling strength as well as stress concentration around openings.

e) Stress concentration at shell corners

The shell corner shown in Fig.- 3 is of circular shape. The corner shape in Fig.- 4 includes a right angle whose stress concentration was taken into account for the gate leaf design [11) p301].

3. Analysis based upon bending- torsion theory

3. 1 Stress distribution on a gate section

In addition to various stresses described in the preceding chapter for simple torsion theory, shearing stress τ_w due to bending- torsion moment T_w and normal stress σ_z which results from warp's derivative dw/ dz appear in this theory. Integration of σ_z over a gate section is zero in any case. Distribution on a gate section of τ_w is defined by shear flow of bending- torsion and that of σ_z is defined by Ψ. Fig.- 32 shows shear flow and warping function of same section as Fig.- 8. Difference between shear flows of bending- torsion and bending is quite clear. As seen in the shear flow due to simple torsion, the shear flow due to bending- torsion is composed of a moment around a sectional shear center. The sum of T_w and T_s which are sectional torsion moments corresponding to bending- torsion and simple torsion respectively is in equilibrium with external torsion moments. Ratio of T_w to total torsion moment indicates intensity of effect from bending- torsion but, <u>even in the case of $T_w = T_s$, τ_w will be remarkably greater than τ_s because shear flow due to simple torsion is constant over a gate section whereas that of bending- torsion changes between plus and minus values</u>. This is

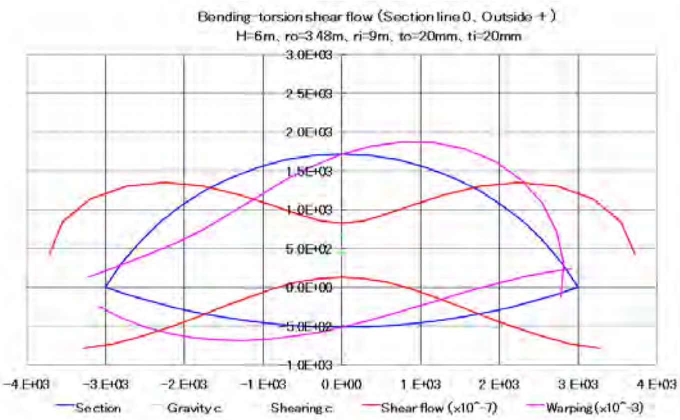


Fig.- 32 Fish Belly Shape

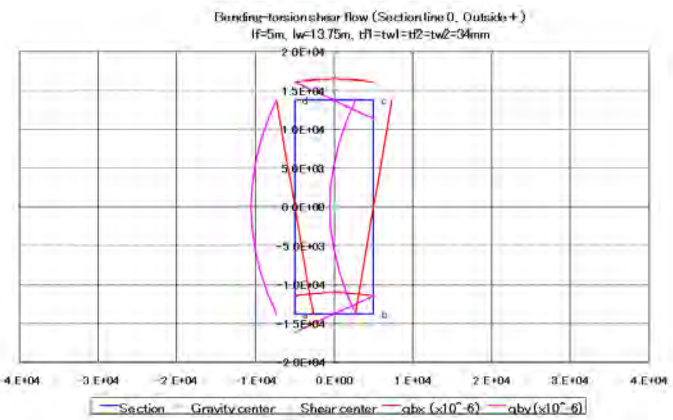


Fig.- 33 Rectangular Shape

the reason why bending- torsion greatly disturbs sectional stress distribution which will be shown later. Fig.- 33 shows results of calculation of the same section as Fig.- 9.

3．2 Analysis by elastic equations

a) Analytical method

The same analytical model and external loads as for simple torsion theory is taken in analysis. The formulas of deformation due to concentrated torsion moment are replaced by formulas including bending- torsion in addition to simple torsion. Distortion angle Θ is obtained by solving following basic equation of bending- torsion with condition of load

$$EC_{bd}\frac{d^4\Theta}{dz^4} - GJ_t\frac{d^2\Theta}{dz^2} - m_t = 0 \qquad \cdots\cdots(16)$$

and boundary shown in Fig.- 12. EC_{bd} in the equation denotes bending- torsion rigidity, GJ_t denotes simple torsion rigidity and m_t denotes external torsion moment. Formula (17) is a resolution of the equation for a sectionally homogeneous structure [10]. Using this formula we can derive formulas (18) and (19) which replace formula (2). An elastic equation is obtained in the same manner as simple torsion theory.

$$\Theta = \frac{T}{EC_{bd}}\left[\frac{z}{a^2} - \frac{sh\{a(l-c)\}sh(az)}{a^3 sh(al)}\right],\ \text{for } z \geqq c\quad \frac{T}{EC_{bd}}\left[\frac{c}{a^2} - \frac{sh\{a(l-c)\}sh(az)}{a^3 sh(al)} + \frac{sh\{a(z-c)\}}{a^3}\right] \qquad \cdots\cdots(17)$$

where $a = \sqrt{GJ_t \div EC_{bd}}$, c : z=c is load point.

[Rotation angle at j support due to m_i on i section]

$$\theta_{ij} = \left\{kj - \frac{sh(k[n-i])sh(kj)}{sh(kn)}\right\}\frac{m_i}{k^3 EC_{bd}},\ \text{for } j>i\quad \left\{ki + sh(k[j-i]) - \frac{sh(k[n-i])sh(kj)}{sh(kn)}\right\}\frac{m_i}{k^3 EC_{bd}} \qquad \cdots\cdots(18)$$

where $k = al_s$

[Displacement at j support due to m_i on i section]

$$\xi_{tij} = \theta_{ij} l_{py}、\ \eta_{tij} = \theta_{ij} l_{px} \qquad \cdots\cdots(19)$$

b) Results

The procedure to obtain internal forces from statically indeterminate reactions given as results of the elastic equation is exactly the same as simple torsion theory except that T_w and T_s are obtained separately and the internal torsion moment is the sum of them. In addition to new formulas for T_w and T_s, formulas are necessary to obtain σ_z and θ. The following formulas satisfy all needs above where θ_{kj} is rotation angle of j section due to m_{ek} acting at k section and ' denotes ordinary differentiation. θ, T_{sj} and T_{wj} vary along bottom support interval and they can be obtained by handling the j

[External torsion moment acting on i section]

$$m_{ei} = m_{si} + X_i l_{py} + Y_i l_{px} \qquad \cdots\cdots(20)$$

[Rotation angle at i section and its derivatives]

$$\theta_j = \sum_{k=0}^{n}\theta_{kj}、\ \theta'_j = \sum_{k=0}^{n}\theta'_{kj}、\ \theta''_j = \sum_{k=0}^{n}\theta''_{kj}、\ \theta'''_j = \sum_{k=0}^{n}\theta'''_{kj} \qquad \cdots\cdots(21)\sim(24)$$

[Simple torsion moment at j section]

$$T_{sj} = GJ_t\,\theta'_j \qquad \cdots\cdots(25)$$

[Bending-torsion moment at j section]

$$T_{wj} = -EC_{bd}\,\theta'''_j \qquad \cdots\cdots(26)$$

in the formulas as a real number. Following examples correspond to the cases shown in the simple torsion theory and their numbers are as same as the corresponding case.

[Example 1]

Fig.- 34 ～37 shows the results of analysis. Fig.- 34 shows θ and its derivatives. The marks x in the figure indicate θ of simple torsion theory. The lateral axis represents section numbers. Vertical axis represents calculated results after multiplied by rates shown. θ is in a rather smooth curve and exactly agrees with simple torsion theory at free end of the gate. θ` is in proportion to T_s, which is apparently different from simple torsion theory. θ`` is in proportion to σ_z and approximately periodic except for both terminal compartments. θ``` is in proportion to T_w and shows same periodicity as θ`` except that its sign is in reverse on both sides of the section including a bottom support. It is estimated that the reversals resulted from abrupt changes in direction of sectional warp change. Fig.- 35 shows support reaction, shearing force and bending moment, and they are almost similar to simple torsion theory because there is almost no difference between statically indeterminate reactions of simple and bending torsion theories. Fig.- 36 shows torsion moment. T_s and T_w vary between bottom supports but their sum is constant and equals the internal torsion moment calculated from external forces. <u>This indicates that the mean amplitude of bending- torsion moment is controlled by the number of bottom supports rather than the sectional particulars. In short, it is estimated that the greater the number of bottom supports, the less the mean amplitude</u>

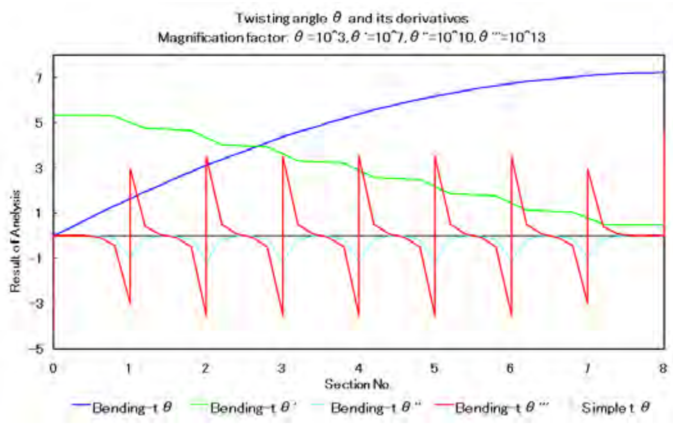


Fig.- 34 θ and its Derivatives

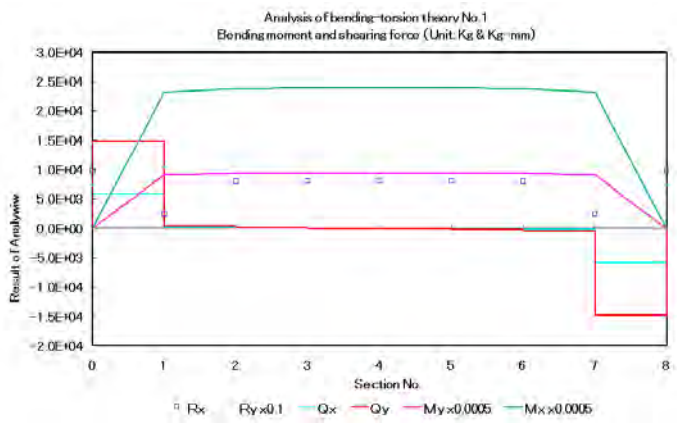


Fig.- 35 Bending Moment etc.

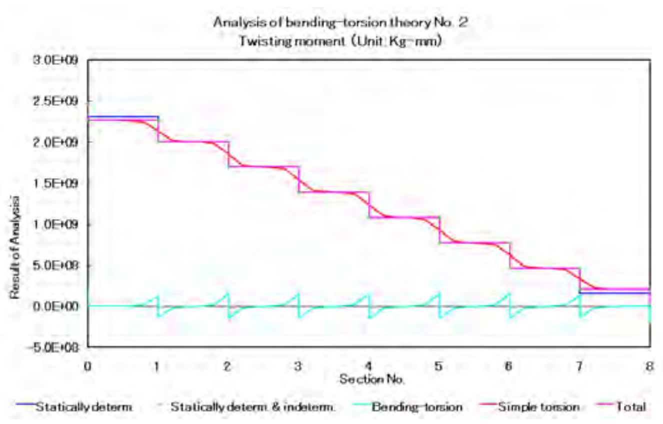


Fig.- 36 torsion Moment

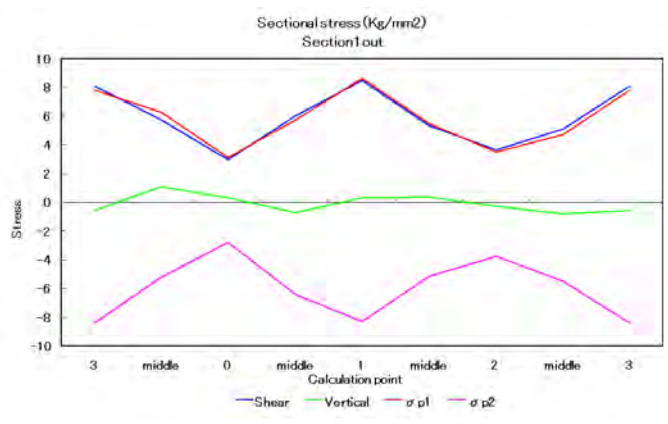


Fig.- 37 Stress

of bending- torsion moment. Fig.- 37 shows shearing stress, normal stress and principal stress just outside (left hand side) of section 1. The degree of stress distribution disturbance due to bending- torsion will become quite clear when these stresses are compared to Fig.- 20. It has been confirmed that disturbance of stress just inside of section 1 is at the same level but in the reverse direction and that the disturbance just inside section 0 is negligible because the effect of bending- torsion there is very small. Even if a big disturbance were to exist, the state of stress is still pure shear. Causes of the disturbance are τ_w and σ_z and its intensity is much greater than disturbance by stresses due to bending. It is noticeable that the effect of T_w on stresses is very large although T_w is very small in comparison to T_s. Effect of bending- torsion is more remarkable at middle or free terminal section because the moment of simple torsion decreases towards the free terminal whereas moment of bending- torsion varies periodically with almost a constant amplitude throughout the gate length. In case of a small scale gate, neglect of bending- torsion will have no effect on the strength problem because the middle or free terminal portion of the gate usually has a big strength margin. In case of super large scale gates, a design without considering bending- torsion cannot hold good because the greater portion of the gate body is in a critical state of stress due to decrease in plate thickness for economic reason toward free terminal. Bending- torsion effects sectional stress distribution much more than sectional forces.

[Example 3]

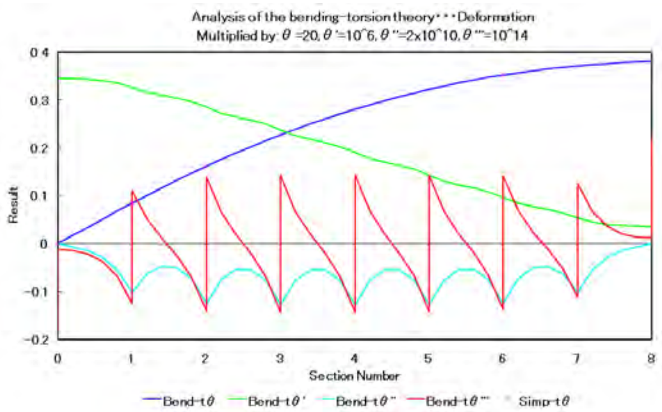


Fig.- 38 θ and its Derivatives

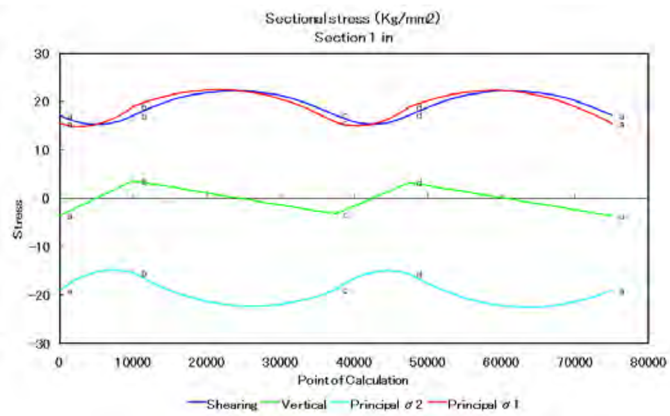


Fig.- 39 Stress

Fig.- 38 shows deformations. General tendency including internal forces is the same as Example 1 except that there is a little difference in details. Curves of θ`` and θ``` between bottom supports of Example 1 have a tendency of keeping it close to the 0 axis, whereas those of Example 3 deviate from it. It has been confirmed that this different tendency mainly stems from a difference of k values in both examples (k=9.73 in Example 1 and 3.07 in Example 3). Fig.- 39 shows shearing stress, normal stress and principal stress just inside section 1 and the disturbances are much less than Example 1. The difference in disturbances of both examples is estimated to come from difference of sectional shapes because the number of compartments in both cases are same and the ratios of T_w and T_s on corresponding sections in both cases are supposed to be nearly equal. In short, rectangular shape can resist bending- torsion with higher efficiency than fish belly shape.

3. 3 Analysis by finite element method

A super large scale gate shown in Fig.- 5 has been analyzed by the finite element method, whose importance in analysis of torsion type structures is described referring to this gate as a subject matter.

a) Similarity of deformation

Finite element methods using various kinds of elements are now available for practical use and analyzed result by them shows good similarity to measured deformation at site as well as model experiments as long as a proper mesh is chosen for nature of analyzed structure and purpose of analysis [3). If the concept of bending- torsion theory accords with the natural phenomena, this will be expected to appear in the results of analysis by the finite element method.

b) Results

An explanation is given according to the reference [5). The gate body is structurally divided at its center and top and bottom of the gate terminal are supported. The gate has a rectangular section, whose shell plate is stiffened by auxiliary vertical girders, supported by horizontal girders, which in return are supported by bulkheads, whose bottoms rest on rollers. Analysis was carried out by IBM 9021 using "NASTRAN" which is a computer software developed at NASA. The degree of freedom at each joint of elements is 6 and elements can represent beam members also. The whole gate body was replaced by about 2500 plate elements and about 2000 beam elements. Water pressure acts on the shell plate but no partial bending arises on the plate elements because all joints of plate elements are located on beam elements. Fig.- 40 shows an example of the results and gives the magnitude and direction of principal stress on the shell plate. Stress on whole gate body is almost in a state of pure shear.

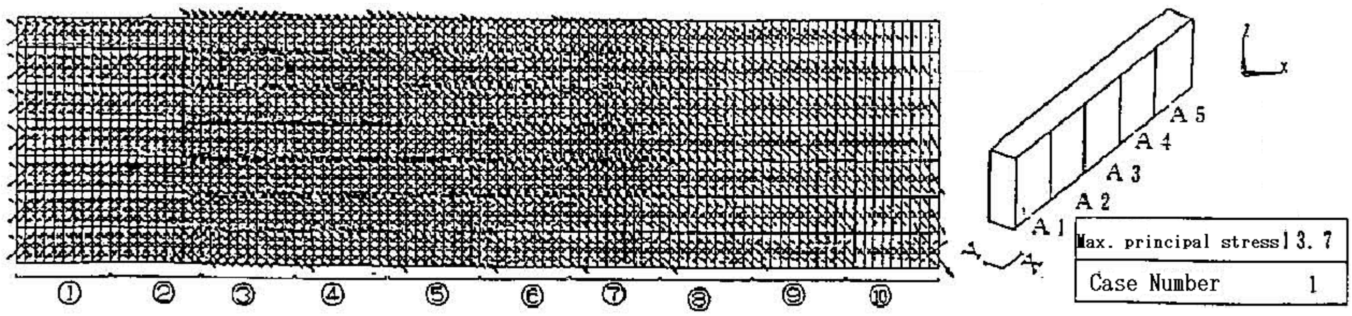


Fig.- 40 Distribution of Principal Stress

c) Comparison of sectional stress

The analytical result by the finite method is compared with the stress obtained through elastic equations. For this comparison, stress on all surfaces of the gate is necessary. New analysis by finite element method was carried out with exactly the same structural members and meshing as shown in the reference using the same program software but by IBM 750 personal computer and it was confirmed that principal stresses on the pressurized surface agree with the values given in Fig.- 40. The calculated stress on a plate element corresponds to value at center of an element and all stress comparison was made on sections cut through element centers except that stress in the intermediate section between roller supports was deemed to be equal to mean value of elements which are located on both sides of the section because the section coincides with the element boundaries. Fig- 41 shows plate thickness of gate section and its number. Plate thickness of each section is constant. The compartments marked by inclined lines in the figure were selected for stress comparison because of the consideration that elastic equations are applicable only to a sectionally homogeneous structure. The comparison was made on the left side, middle and right side sections of each compartment. Fig.- 9 and 33 correspond to section 0 ～ 2 except that no auxiliary members such as vertical and horizontal girders are included. Fig.- 42 shows principal stress σ_1 and σ_2 of nine sections analyzed by the finite element method, the elastic equation method on bending- torsion theory and the elastic equation method on simple torsion theory which are shown in the figure by marks "finite", "bending" and "simple" respectively. The "inside" or "outside" in the figure refers to whether the section shown is located on the gate center line side (see fig.- 41) or the opposite side respectively, and a ～ d represent points on a section and corresponds to those in Fig.- 9 or 33. Bending- torsion theory is much more explanatory of the results by the finite element method than simple torsion theory. The results by the finite element method and by bending- torsion theory are supposed to agree with each other but detailed observation of them disclose fairly large discrepancies on the outside of section 2 and sections between 8 and 9. It is estimated that the discrepancies mainly come from the analytical model for the elastic equation. In the model, deformation of vertical gate sections was not considered, distribution load was replaced by concentrated loads on web frames and discontinuity in plate thickness over gate length was not considered. It would be quite expectable if deformation of vertical sections and auxiliary members appears strongly on section 8 ～ 9 whose rigidity are much less and if the effect of the thickness discontinuity appears noticeably on section 2 where a big jump in plate thickness exists. It cannot be denied that bending- torsion has an important effect on stress distribution even if there is a regardless of lack of uniformity in the comparison and it will be concluded that analysis by finite element method is inevitable in design of super large scale torsion type structures as long as elastic equations are not applicable for sectionally variable structures.

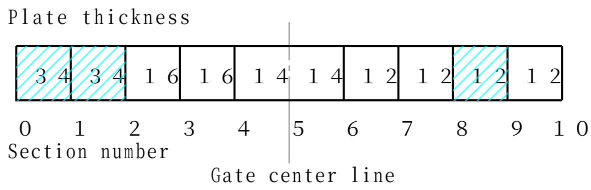


Fig.- 41 Shell Thickness Variation

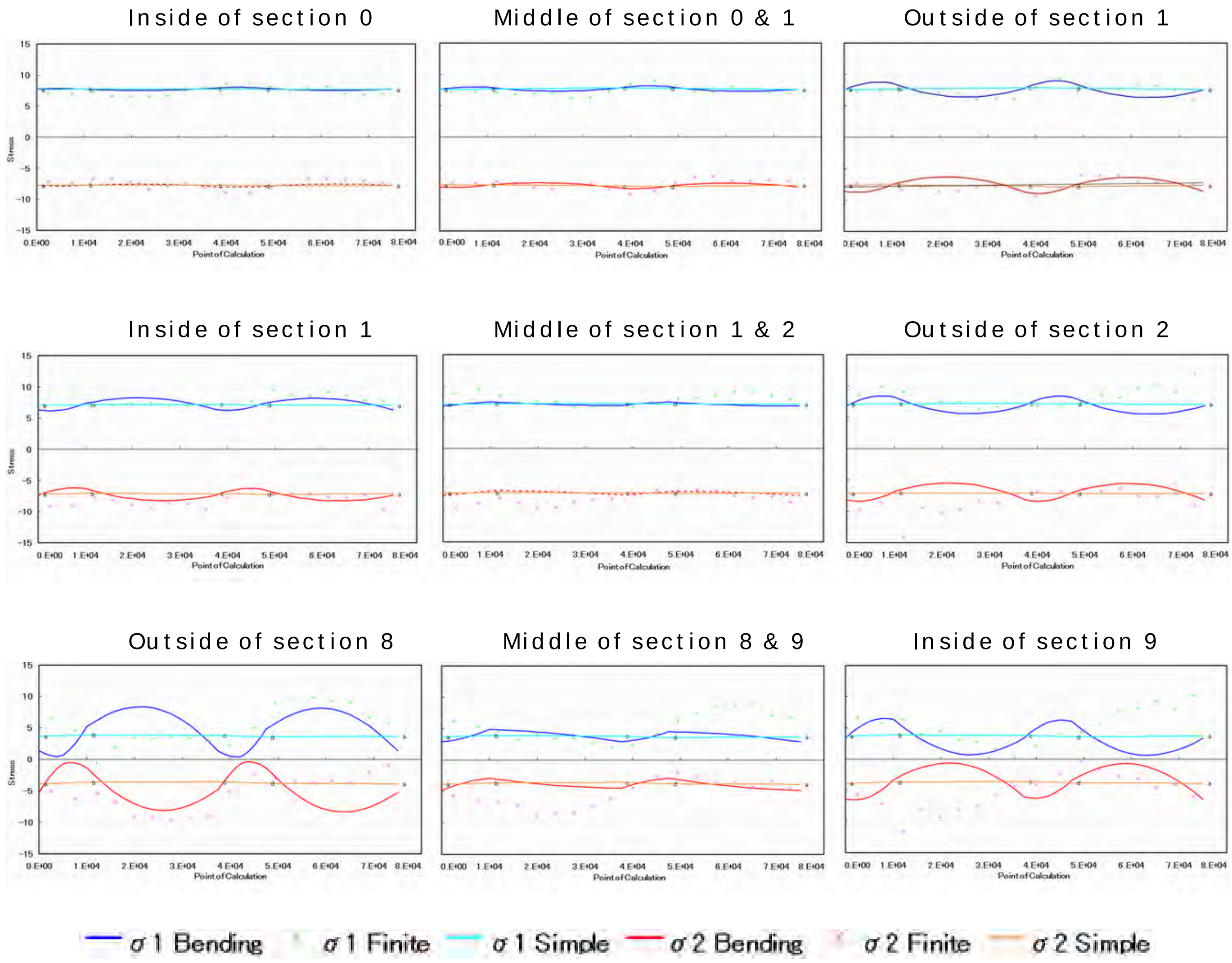


Fig.- 42 Principal Stress on Sections (Kgf/mm^2)

4. Simplified analytical method

The analytical method described in chapters 2. and 3. are all very tedious in their execution and not suitable for use in analyzing many small scale gates in a short time. There are two ways of simplifying the analysis. In both ways the rigidity in bending-torsion is assumed to be zero. Case 1: Bending rigidity is assumed to be zero, and deformation and stress are found directly from statical torsion moment with the help of torsion rigidity and shear flow only, and computation can be made very easily. Case 2: Bottom support is distributed continuously, and the condition of equilibrium for torsion type structure is expressed by a differential equation which has been solved for sectionally homogeneous structures [8]. A more simple solution has been also found for the case in which a sectional gravity center coincides with a shear center [9]. Computation of case 2 is just a bit more complicated than case 1.

5. Conclusion

The result of study can be summarized as follows.

(1) The shearing stress due to bending-torsion moment is strikingly bigger than one due to simple torsion moment of the same magnitude. Bending-torsion moment is comparatively small but its eventual effect on sectional stress can not be deemed negligible in torsion type structures.

(2) Bending-torsion appears throughout over the gate with almost the same amplitud and period, and its effect on sectional stress is more remarkable in the vicinity of non-supported gate end where torsion moment is comparatively small. Gate design without considering bending-torsion cannot hold good in the case of super large scale torsion type gates whose most of the surface is in a critical state of stress and stress due to bending-torsion is dominant over a large portion of the gate.

(3) Table-2 shows the applicable field of analytical methods. The simplified analytical method is applicable to small gates with homogeneous sections, elastic equation method by simple torsion theory and space frame method are used for large gates with non-homogeneous sections and the finite element method is applicable to super large scale gates. Formulas for Θ which can replace formula (17) and are applicable to sectionally non-homogeneous gates would be required if elastic equations of bending-torsion theory are applied to super large scale gates. To define quantitative boundaries of applicable gate scales, it is estimated that a large number of structural analyses would be required to systematically grasp the effects of bending rigidity, sectional discontinuity and bending-torsion rigidity.

(4) There is a possibility for torsion type structures to be applied to super large scale structure including bridges etc.

		Analytical theory	
		Simple Torsion	Bending-torsion
Gate section	Uniform	Elastic equation Space frame method Symplified analysis	Elasitc equation Finite element method
	Variable	Elastic equation Space frame method	Finite element method

Table-2 Aplication Range

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捩り構造ゲートの解析方法

寺田 溥[*1]

単純捩り理論、曲げ捩り理論、及び、簡易法に分けて構造解析の方法を示し、各方法の関係を明かにした。曲げ捩りの影響は応力に強く現れ、その程度は有限要素法で把握できる。超大型ゲートの設計は曲げ捩りを無視して成り立たない。捩り構造は幾つかの本質的優位性を持っているが、普及率は高くない。解析方法の複雑さがその背景にある。

キーワード：ゲート、捩り、曲げ捩り、構造解析、薄肉閉断面

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