PowerPoint 版

Torsion Type Hydraulic Gates

Mitigation of Warping & Optimum Design

TeraMatsu

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1. Introduction

A torsion type closed thin shell section is overwhelmingly superior in structural functions to a bending type structure because the torsion type resists against external load with second power of the closed area whereas the bending type resists with the sectional moment of inertia of thin shell, and this superiority gets more remarkable as the structural span becomes longer.

We have two theories in structural torsion, the simple torsion and the bending-torsion. In case of the simple torsion, shearing stress yields on a cross section due to simple torsion moment. In case of the bending-torsion, bending-torsion moment is applied to the section in addition to simple torsion moment. Shearing stress due to simple torsion moment distributes uniformly over the section whereas shearing stress due to bending-torsion moment meanders so that peak value of their total sometimes become much more than 200% of simple torsion alone. Furthermore, vertical stress that is causally related to sectional warping and in equilibrium with shearing stress of bending-torsion arises on the section.

Since stress is in proportion to the product of a form coefficient, deformation and a spring constant that shearing and vertical stresses of bending-torsion can decrease or be eliminated with form coefficient reduction (patent pending). On the other hand, the form coefficient reduction will result in section modulus decrease which may cancel its effect because intensity of deformation is equal to internal force divided by the product of a section modulus and a spring constant.

The most important section modulus controlling deformation of a torsion type structure is J_t. The product of J_t and a spring constant is a sectional rigidity which resists simple torsion moment. Although J_t decreased because of the form coefficient reduction, gate weight cut is possible by making up the lost J_t with help of cross section form change.

Superiority of the torsion type structure to the bending type may be made more certain by applying the form coefficient operation to **optimum design** whose object function is cost.

2． What is bending-torsion?

2． 1 Who started to use the name of "Bending-torsion"?

Use of the name "Bending-torsion" was started by H. Wagner. He established general theory for bending-torsion of H shape shown on Fig.-2-1. In his study, Ｃｂｄ（section modulus for bending-torsion）was called, Bieguugsverdrehungswiderstand（a sectional rigidity of bending-torsion）.

In Fig.-2-1, End A is a support side of the H shape and End B is supposed to be twisted around X axis by θ due to external torsion moment and section 1234 of End B warps and gets to 1'2'3'4'. The warping will change along X axis if any restriction or change in torsion moment exists between A and B. Existence of change in warping means each flange of H shape bends in its own plane so that a pair of shear forces exists on the flange sections as shown on Fig.-2-2 and it forms a couple. This couple together with simple torsion moment is in balance with external torsion moment.

It is reported that "Bending-torsion" was named after above explained structural phenomena. The couple is called bending-torsion moment. There are many cases which give different images from such as the original explanation since study in this field spread but still this name is surviving.

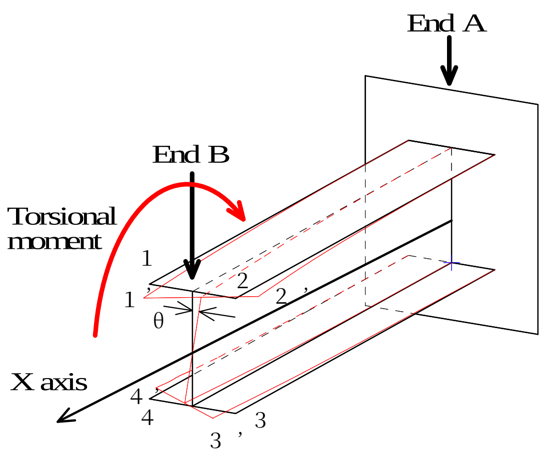


Fig.-2-1 Origin of "Bending-torsion"

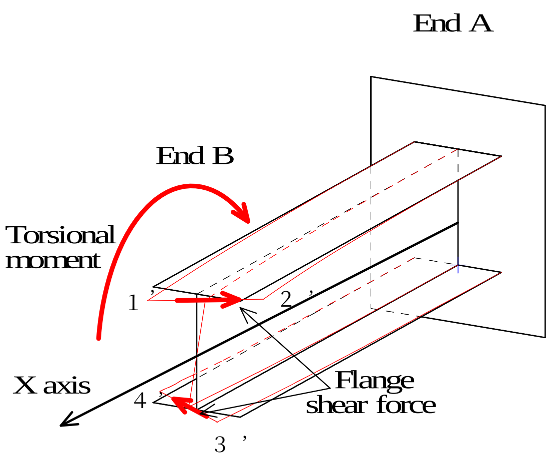


Fig.-2-2 Flange shearing stress

2． 2 Simple torsion and bending-torsion

Explanation of difference between the simple torsion and the bending-torsion is presented on a closed section of shin shell as an example that has a different image of "Bending-torsion".

（1）The simple torsion

Fig.-2-3 shows an example of deformation due to the simple torsion of a closed shin shell section. End A is a support side and End B is a loading side of the closed section, which is twisted around X axis due to torsion moment applied on End B and section 1234 of End B is transformed to 1'2'3'4' without sectional warping. The simple torsion is a twisting without sectional warping. It is also called uniform twisting. It includes not only twisting without warping（for instance, round section）but also twisting having warping but no bending-torsion and twisting whose bending-torsion is negligible. Most of twisting is deemed to belong to this category and all of them are handled as if they have no warping. Internal forces on a cross section of simple torsion are simple torsion moment, bending moment and shearing force in general.

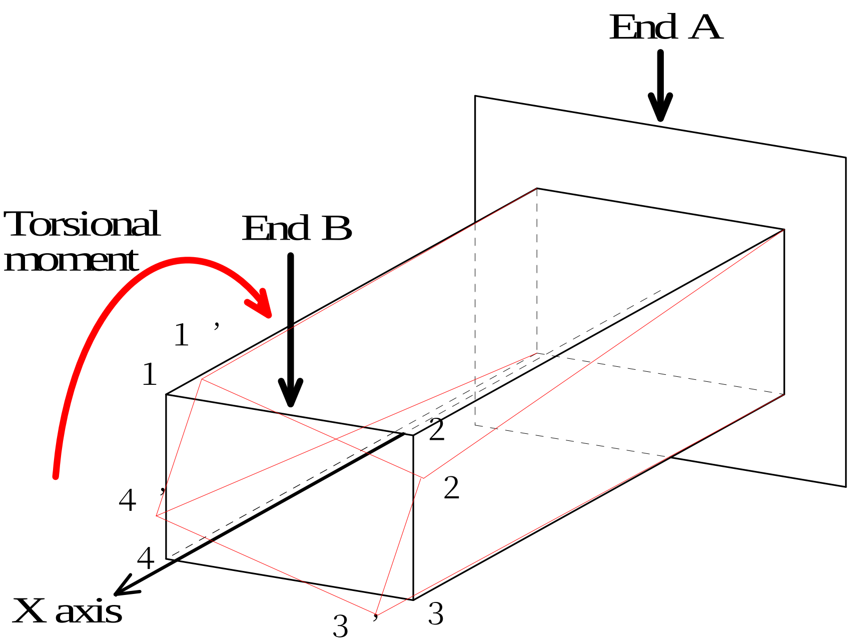


Fig.-2-3 Simple torsion of a closed section of thin shell

（2）The bending-torsion

Fig.-2-4 shows deformation of a rectangular closed section of thin shell in bending-torsion. End A is a support side and End B is supposed to be twisted around X axis due to external torsion moment and section 1234 of End B warps and gets to 1'2'3'4'. The warping will change along X axis due to restrictions or change in torsion moment existing between A and B. As is seen on Fig.-2-2, shearing forces yield on the bottom and the top plate and on the both side plates and they form two pairs of coupling. These couples together with simple torsion moment are in balance with external torsion moment. In addition to the three internal forces of simple torsion, two other items, bending-torsion moment and vertical stress of sectional warping, exist on the cross section in case of the bending-torsion.

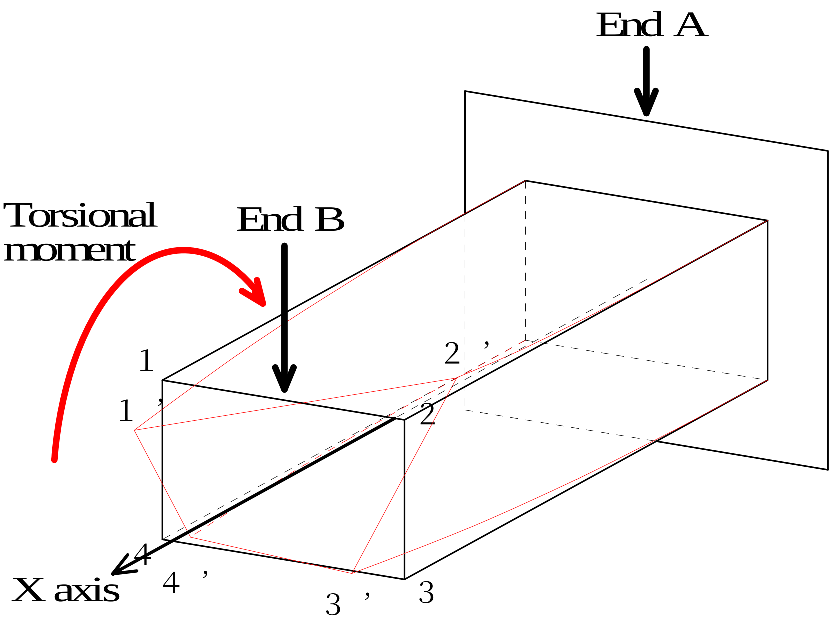


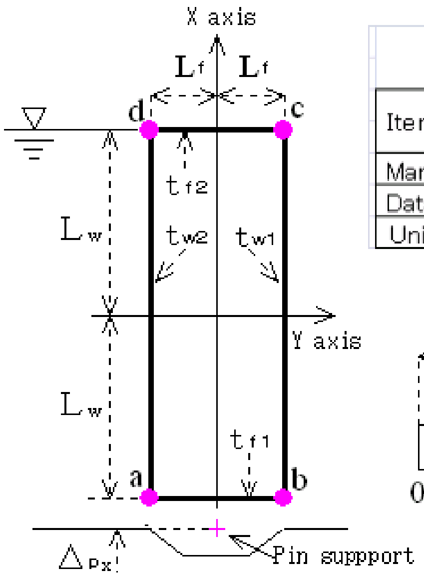
Fig.-2-4 Bending-torsion of a closed section of thin shell

3. Effect of the bending-torsion

Although there is no big change in internal forces, the shearing stress which distributes uniformly over a section in case of simple torsion will meanders due to existence of bending-torsion moment and its peak value increase to a remarkable extent sometimes that is shown in following examples. The elastic equation method shown on bibliography (6) was applied to the structural analysis of the examples.

3. 1 Effect on internal force

Fig.-3-1 is Fundamental case of a rectangular section, on which it will be shown that no big change due to the bending-torsion occurs in internal forces of simple torsion. 0 of the front view in the figure is a support end and 8 is a free end.



Fundamental case of a rectangular section

Item	Form		Thickness				Width	Pin support		
								Location		Q.
Mark	Lf	Lw	tf1	tw1	tf2	tw2	Lg	Lpy	Δpx	n
Data	5000	13750	34	34	34	34	100000	0	2000	9
Unit	mm									

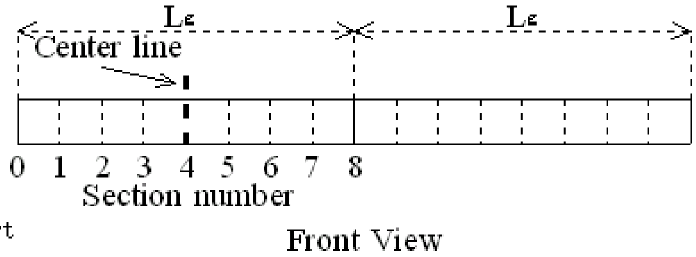


Fig.-3-1 Fundamental case of a rectangular section

Fig.-3-2 and Fig.-3-3 are internal force analyzed by the simple torsion theory and Fig.-3-4 thru Fig.-3-6 are internal force and deformation analyzed by the bending-torsion theory. Deformation of the simple torsion theory is shown in Fig.-3-4 for the bending-torsion theory. My which is bending moment around Y axis and Qx which is shearing force in X direction are zero because the pin support in Fig.-3-1 locates on X axis on which shearing center is also.

Change in cross sectional internal forces due to additional bending-torsion is as

follows.
(1) Internal torsion moment has small difference, but no change at support terminal.
(2) Sum of simple and bending torsion moments is equal to internal torsion moment.
(3) Small decreases in bending moment and shearing force on the section.
(4) Twisting angle has small decrease, but no change at free terminal.

(1) Analysis of the simple torsion theory

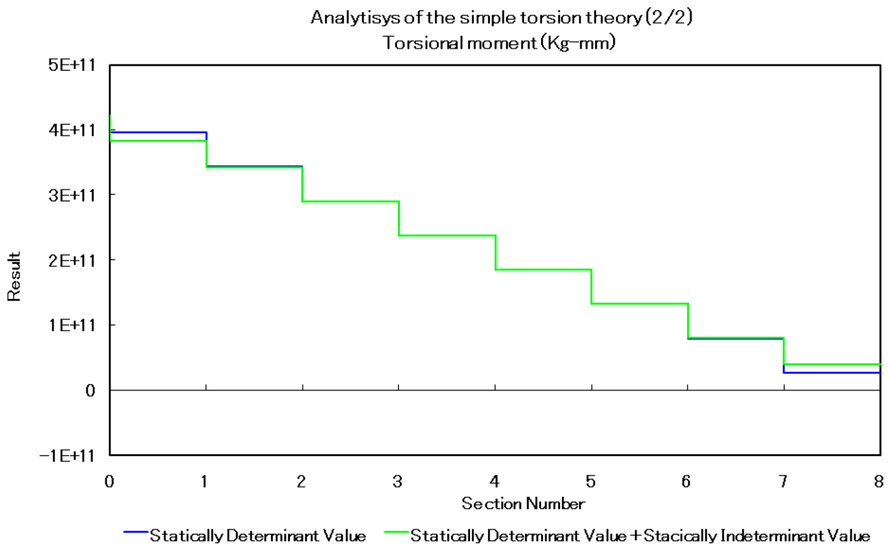


Fig.-3-2 Analysis of the simple torsion theory・・torsion moment

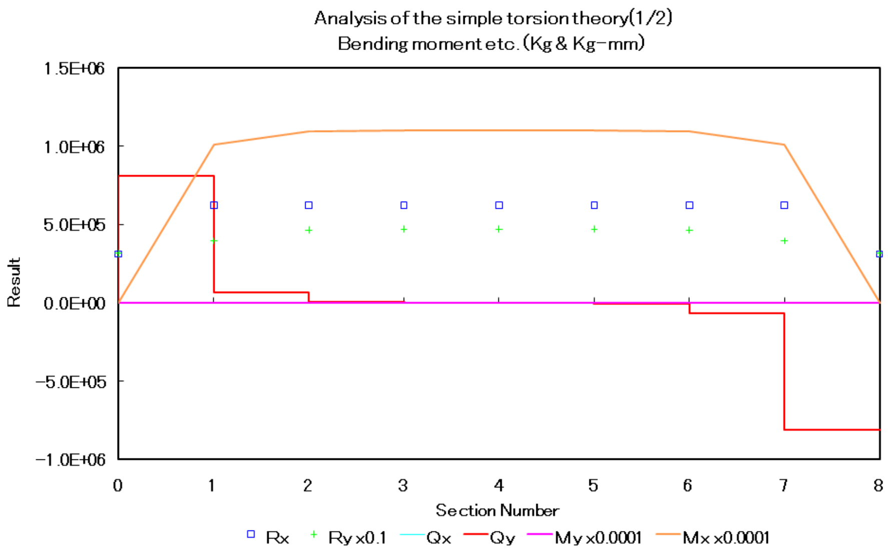


Fig.-3-3 Analysis of the simple torsion theory・・Bending moment etc.

(2) Analysis of the bending-torsion theory

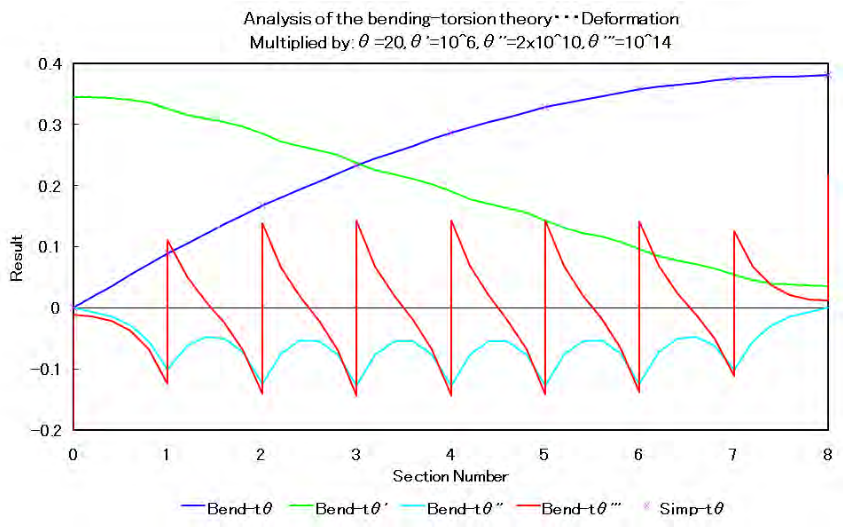


Fig.-3-4 Analysis of the bending-torsion theory・・Deformation

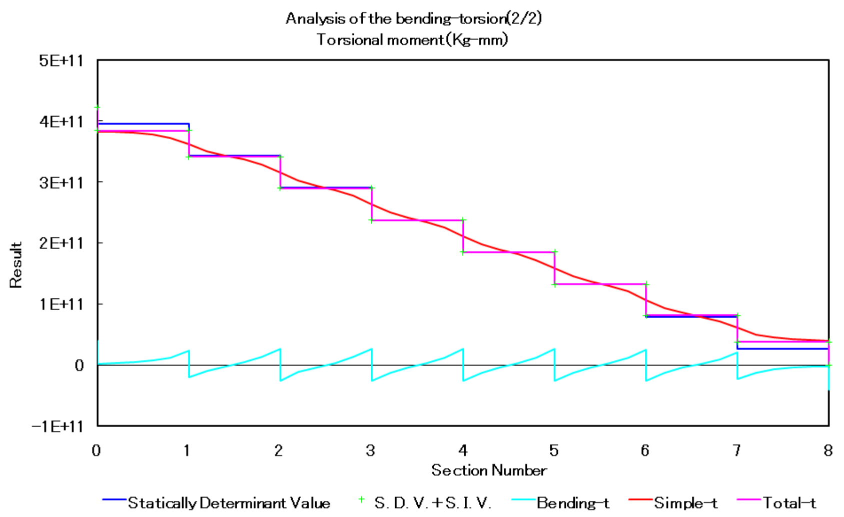


Fig.-3-5 Analysis of the bending-torsion theory・・torsion moment

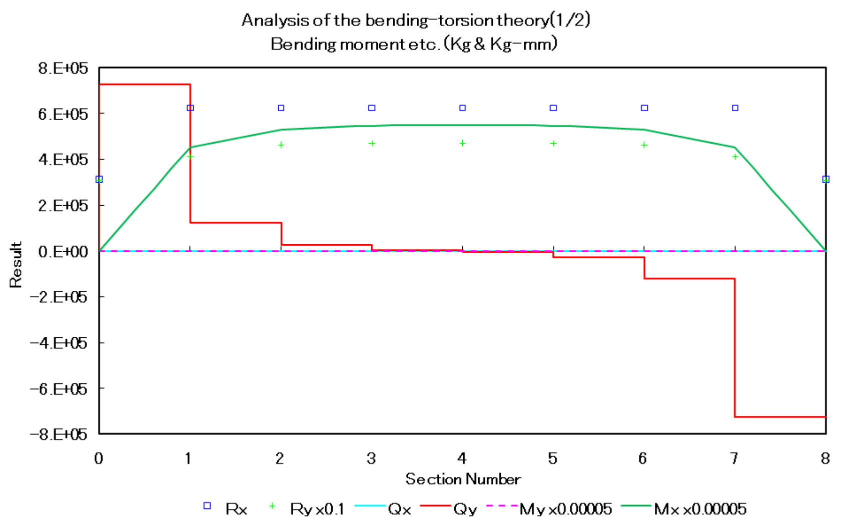


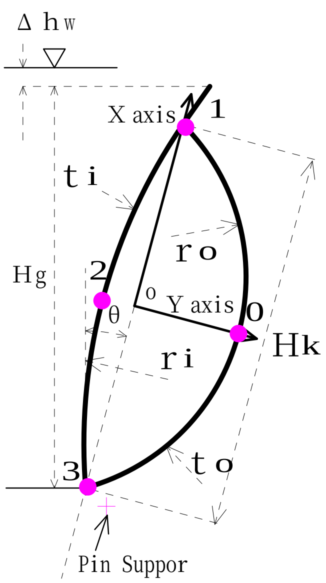
Fig.-3-6 Analysis of the bending-torsion theory・・Bending moment etc.

3．2 Effect on stress

It is observed on both the fish belly section and the rectangular section that the shearing stress which distributes uniformly over a section of the simple torsion will meanders due to additional bending-torsion moment and its peak value increases to a remarkable extent. The section whose stress is shown is identified by, for example, 1out or 1in where 1 indicates section number and out or in indicates outside or inside of the section with respect to a center line of gate. Stresses of both side of a section are presented since sign of bending-torsion moment turns over at the section. The section number and the center line are shown on a corresponding figure.

3．2．1 Stress on a fish belly section

A fish belly section is often utilized for reservoir or river flaps. Fig.-3-7 shows its example. Section Number 0 on the figure is a support end and that of 8 is a free end.



Example of fish belly section

Item	Gate hight	Gate width	Section					Flow depth	Pin support Location		Q	Inclin ation
Mark	Hg	Lg	Hk	ro	ri	to	ti	Δhw	Lpy	Lpx	n	θ
Data	6400	25000	6000	3480	9000	20	20	300	391	3206	9	15
Unit	mm											°

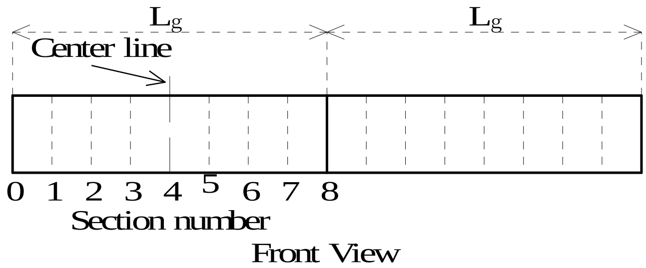


Fig.-3-7 Example of a fish belly section

Fig.-3-8 shows sectional stress of the simple torsion theory and Fig.-3-9 shows that of the bending-torsion theory. Lateral axis on the graphs is girth length along the section shown on Fig.-3-7 and the numbers 0, 1, 2 and 3 on the axis correspond to pink colored points on the figure.

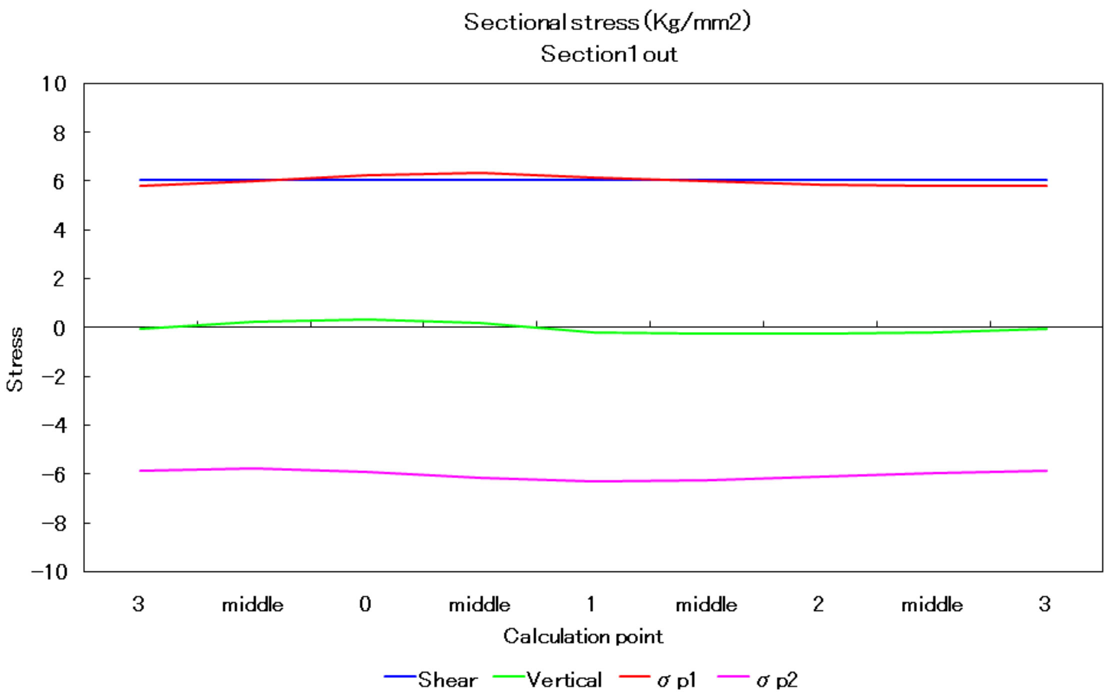


Fig.-3-8 Sectional stress of the simple torsion theory (fish belly section)

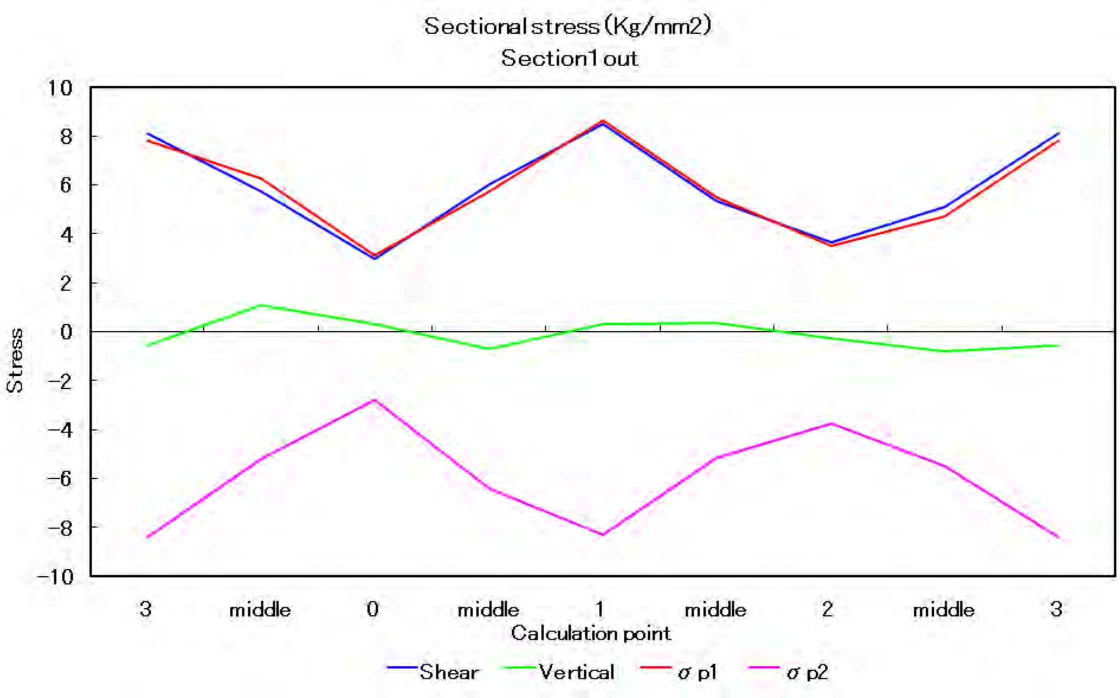


Fig.-3-9 Sectional stress of the bending-torsion theory (fish belly section)

Explanation of Fig.-3-8 (simple torsion): Shear is abbreviation of shearing stress which corresponds to the sum of simple torsion stress and shearing stress relating to bending. Vertical is abbreviation of vertical stress which corresponds the sum of stresses by bending moments around X and Y axes. σp_1 and σp_2 are principal stresses calculated from the shearing stress and the vertical stress. Stress status of the section is almost in pure shearing because absolute value of σp_1 and σp_2 is almost equal to the shearing stress.

Explanation of Fig.-3-9 (bending-torsion): Shear is abbreviation of shearing stress which corresponds to the sum of simple torsion stress, bending-torsion stress and shearing stress relating to bending. Vertical is abbreviation of vertical stress which corresponds the sum of vertical stress by sectional warping and stresses by bending around X and Y axes. σp_1 and σp_2 are principal stresses calculated from the shearing stress and the vertical stress. Stress status of the section is almost in pure shearing because absolute value of σp_1 and σp_2 is almost equal to the shearing stress. Vertical stress by sectional warping corresponds to majority of total vertical stress but is not still dominant among sectional stresses. The more calculation points, the more smooth curves we will get in cases of the shearing stress and the principal stresses. The maximum shearing str**ess** increase rate of bending-torsion versus simple torsion is about 33 %.

３．２．２ Stress on a rectangular section

A rectangular section has been applied to a flap gate of ultra-large ship repair docks. This sectional form is recommendable for a torsion type flood gate of our technical proposal. Two examples are presented for comparison. One shows stress on a section in the vicinity of a support end where internal twisting moment is big and other shows stress on a section in the vicinity of a free end where internal twisting moment is comparatively small.

３．２．２．１ Stress in the vicinity of a support end

Sectional stress in the vicinity of a support end of Fundamental case shown on Fig.-3-1 is presented. Applied twisting moment is maximum at a support end.

（１）Section 1out

Fig.-3-10 shows sectional stress of the simple torsion theory and Fig.-3-11 shows that of the bending-torsion theory. Lateral axis on the graphs is girth length along the section shown on Fig.-3-1 and the sign a, b, c and d in the graph correspond to pink colored points on the figure.

Explanation of Fig.-3-10（simple torsion）: Shear is abbreviation of shearing stress which corresponds to the sum of simple torsion stress and shearing stress relating to bending. Vertical is abbreviation of vertical stress which corresponds the sum of stresses by bending around X and Y axes. σp_1 and σp_2 are principal stresses calculated from the shearing stress and the vertical stress. Stress status of the section is almost in pure shearing because absolute value of σp_1 and σp_2 is almost equal to the shearing stress.

Explanation of Fig.-3-11（bending-torsion）: Shear is abbreviation of shearing stress which corresponds to the sum of simple torsion stress, bending-torsion stress and shearing stress relating to bending. Vertical is abbreviation of vertical stress which corresponds the sum of vertical stress by sectional warping and stresses by bending around X and Y axes. σp_1 and σp_2 are principal stresses calculated from the shearing stress and the vertical stress. Stress status of the section is almost in pure shearing because absolute value of σp_1 and σp_2 is almost equal to the shearing stress. The warping stress which corresponds to majority of the vertical stress may affect sectional stress to certain extent but is not dominant. The maximum shearing stress increase rate of bending-torsion versus simple torsion is about 20 %. Increase of stress occurs between a and b (bottom plate) and c and d (top plate), and decrease occurs between b and c and d and a (side plates).

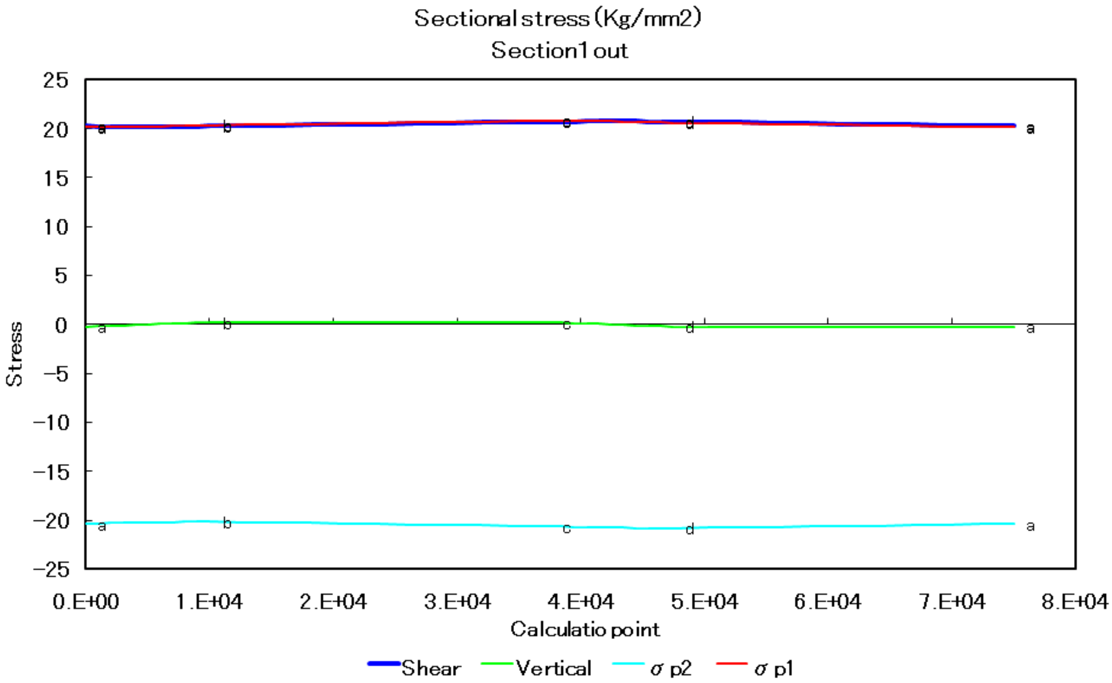


Fig.-3-10 Sectional stress of the simple torsion theory (Rectangular section)

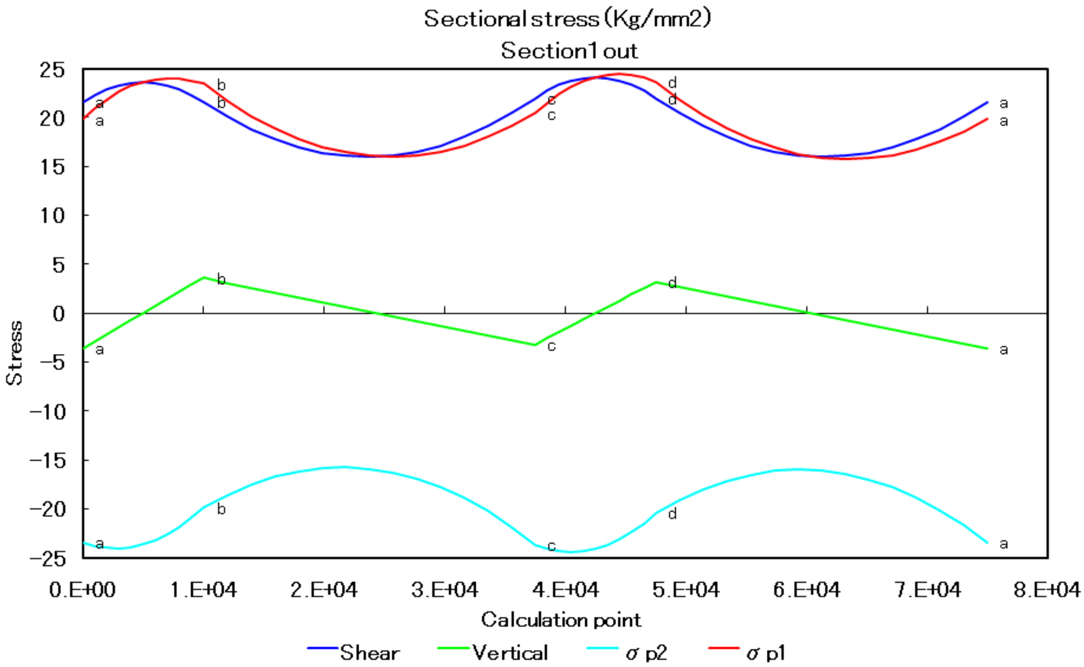


Fig.-3-11 Sectional stress of the bending-torsion theory (Rectangular section)

（2） Section 2in

Fig.-3-12 shows sectional stress of the simple torsion theory and Fig.-3-13 shows that of the bending-torsion theory. Lateral axis on the graphs is girth length along the section shown on Fig.-3-1 and the sign a, b, c and d in the graph correspond to pink colored points on the figure.

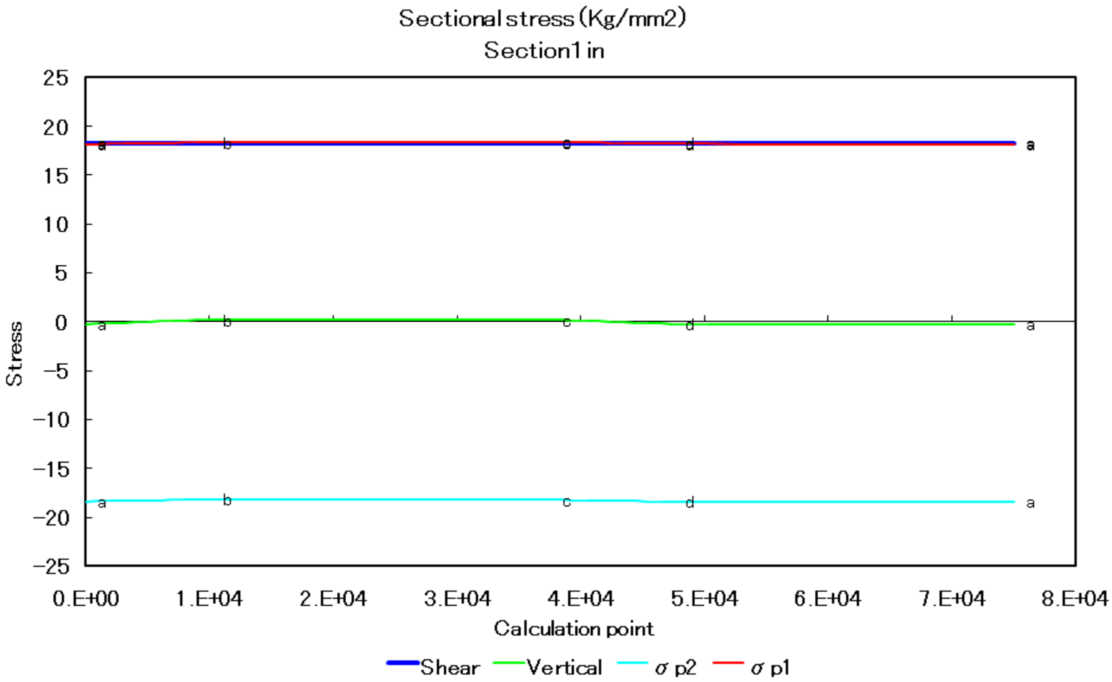


Fig.-3-12 Sectional stress of the simple torsion theory（Rectangular section）

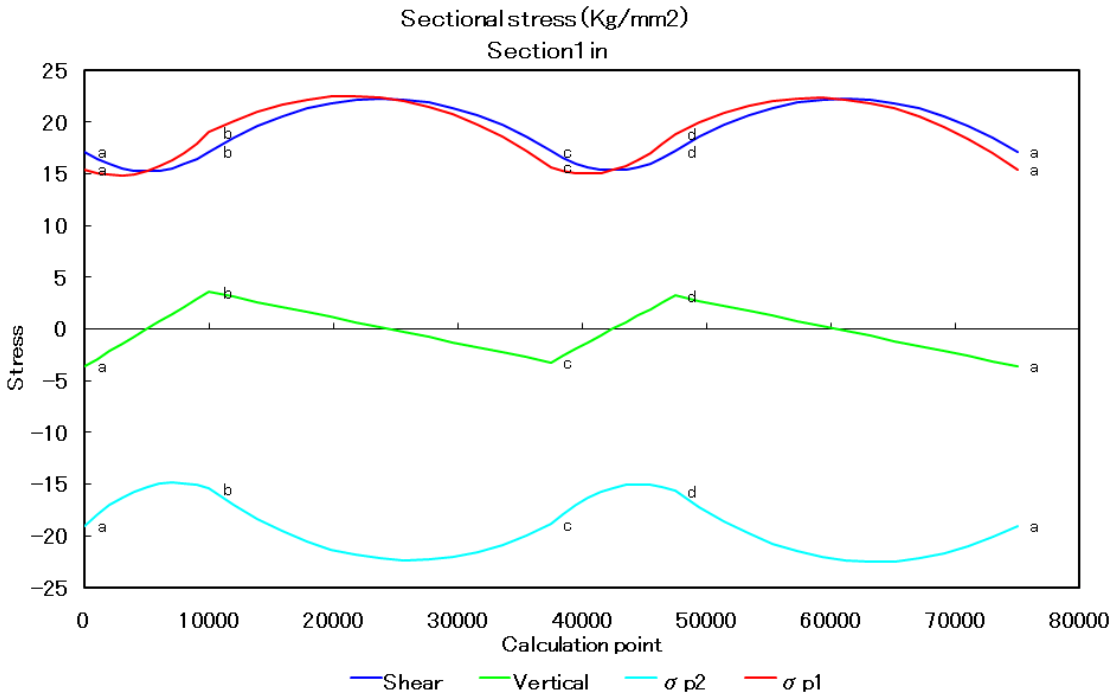
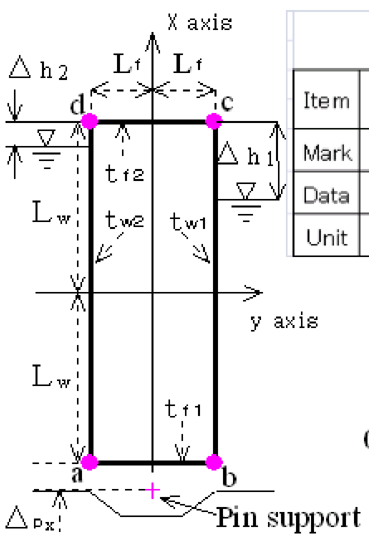


Fig.-3-13 Sectional stress of the bending-torsion theory（Rectangular section）

Explanation of Fig.-3-12 (**simple torsion**): Shear is abbreviation of shearing stress which corresponds to the sum of simple torsion stress and shearing stress relating to bending. Vertical is abbreviation of vertical stress which corresponds the sum of stresses by bending around X and Y axes. σp_1 and σp_2 are principal stresses calculated from the shearing stress and the vertical stress. Stress status of the section is almost in pure shearing because absolute value of σp_1 and σp_2 is almost equal to the shearing stress.

Explanation of Fig.-3-13 (**bending-torsion**): Shear is abbreviation of shearing stress which corresponds to the sum of simple torsion stress, bending-torsion stress and shearing stress relating to bending. Vertical is abbreviation of vertical stress which corresponds the sum of vertical stress by sectional warping and stresses by bending around X and Y axes. σp_1 and σp_2 are principal stresses calculated from the shearing stress and the vertical stress. Stress status of the section is almost in pure shearing because absolute value of σp_1 and σp_2 is almost equal to the shearing stress. The warping stress which corresponds to majority of the vertical stress may affect sectional stress to certain extent but is not dominant. The maximum shearing stress increase rate of bending-torsion versus simple torsion is about 22 %. Increase of stress occurs between b and c and d and a (side plates), and decrease occurs between a and b (bottom plate) and c and d (top).

３．２．２．２ Stress in the vicinity of a free end



Example of a rectangular section

Item	Form		Thickness				Gate width	Pin support Location		Q	Design depth	
Mark	Lf	Lw	tf1	tw1	tf2	tw2	Lg	Lpy	Δpx	n	Δh1	Δh2
Data	5000	13750	Front view				100000	0	2000	11	4.5	0.2
Unit	mm										m	

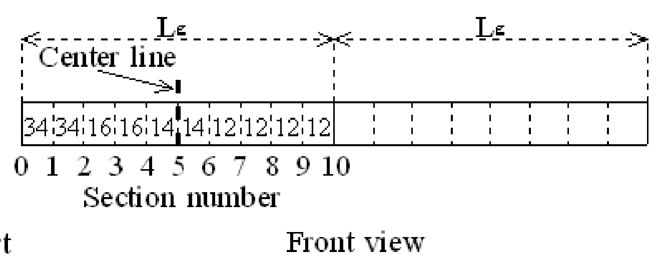


Fig.-3-14 Example of a rectangular section

Sectional stress in the vicinity of a free end of the case shown on Fig.-3-14 is presented. Applied twisting moment at a free end is comparatively small. The section number 0 in the figure is a support end and section number 10 is a free end.

（１）Section 8out

Fig.-3-15 shows sectional stress of the simple torsion theory and Fig.-3-16 shows that of the bending-torsion theory. Lateral axis on the graphs is girth length along the section shown on Fig.-3-14 and the sign a, b, c and d in the graph correspond to pink colored points on the figure.

Explanation of Fig.-3-15（simple torsion）: Shear is abbreviation of shearing stress which corresponds to the sum of simple torsion stress and shearing stress relating to bending. Vertical is abbreviation of vertical stress which corresponds the sum of stresses by to bending around X and Y axes. Resultant is abbreviation of resultant stress which is calculated according to the shearing strain energy theory（Mises-Hencky-Huber-Theory）. σp_1 and σp_2 are principal stresses calculated from the shearing stress and the vertical stress. Stress status of the section is almost in pure shearing because absolute value of σp_1 and σp_2 is almost equal to the shearing stress.

Explanation of Fig.-3-16（bending-torsion）: Shear is abbreviation of shearing stress which corresponds to the sum of simple torsion stress, bending-torsion stress and shearing stress relating to bending. Vertical is abbreviation of vertical stress which corresponds the sum of vertical stress by sectional warping and stresses by to bending around X and Y axes. Resultant is abbreviation of resultant stress which is calculated according to the shearing strain energy theory（Mises-Hencky-Huber-Theory）. σp_1 and σp_2 are principal stresses calculated from the shearing stress and the vertical stress. Stress status of the section is almost in pure shearing because absolute value of σp_1 and σp_2 is almost equal to the shearing stress. The warping stress which corresponds to majority of the vertical stress may affect sectional stress to certain extent but is not dominant. The maximum shearing stress increase rate of bending-torsion versus simple torsion is about 114 %. Increase of stress occurs between b and c and d and a（side plates）, and decrease occurs between a and b（bottom plate）and c and d（top plate）.

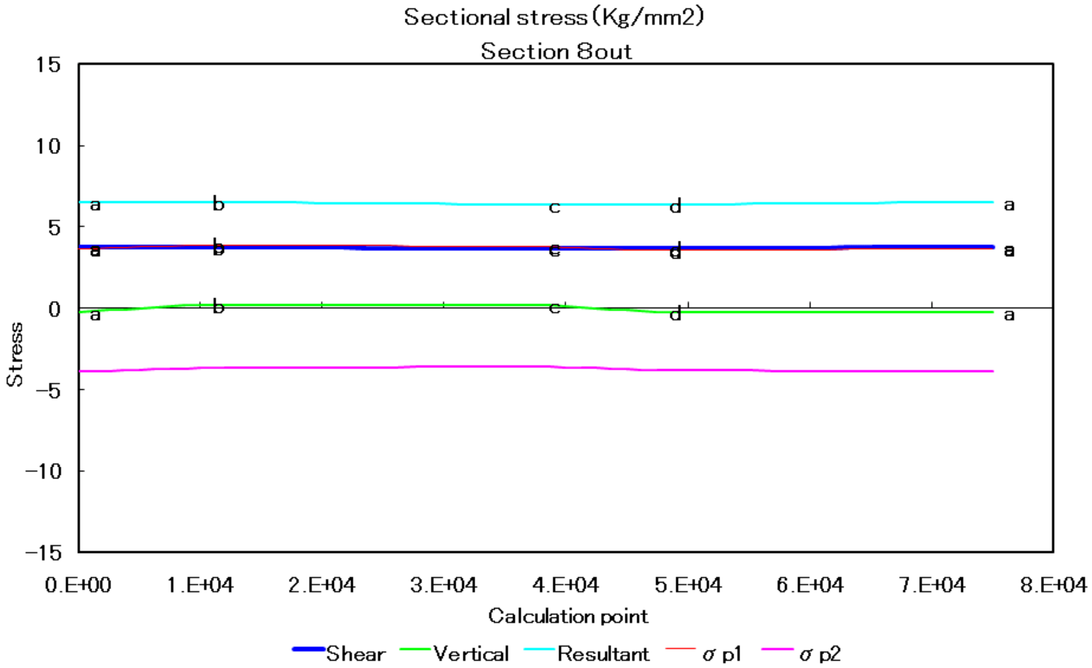


Fig.-3-15 Sectional stress of the simple torsion theory（Rectangular section）

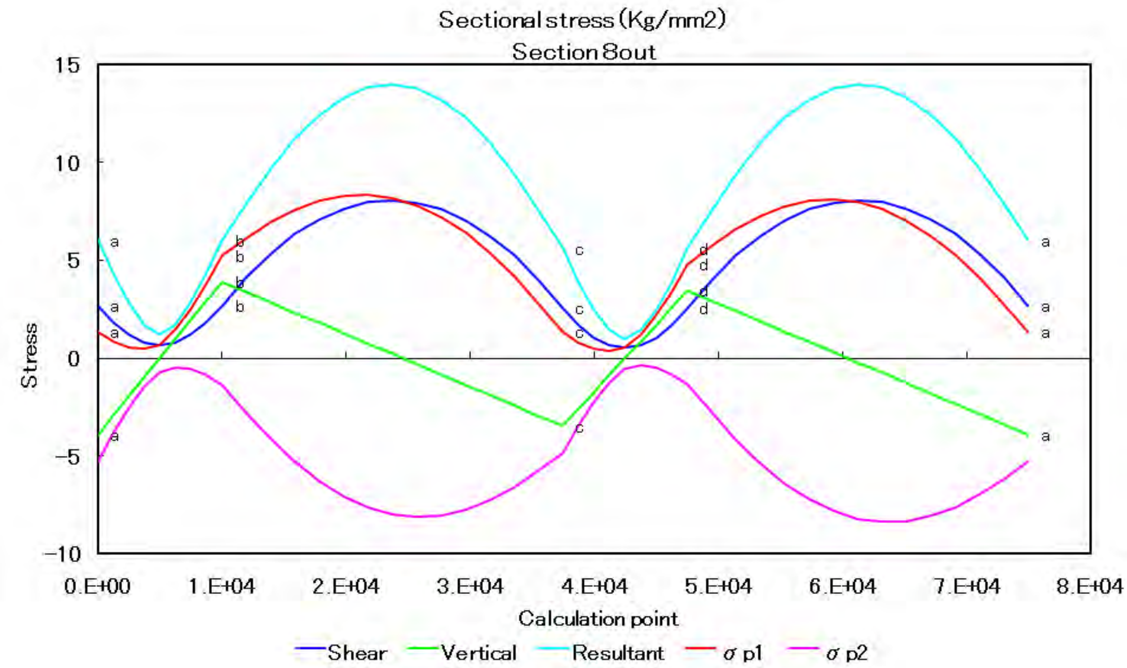


Fig.-3-16 sectional stress of the bending-torsion theory（Rectangular section）

（２）Section 9in

Fig.-3-17 shows sectional stress of the simple torsion theory and Fig.-3-18 shows that of the bending-torsion theory. Lateral axis on the graphs is girth length along the section shown on Fig.-3-14 and the sign a, b, c and d in the graph correspond to pink colored points on the figure.

Explanation of Fig.-3-17（simple torsion）: Shear is abbreviation of shearing stress which corresponds to the sum of simple torsion stress and shearing stress relating to bending. Vertical is abbreviation of vertical stress which corresponds the sum of stresses by bending around X and Y axes. Resultant is abbreviation of resultant stress which is calculated according to the shearing strain energy theory（Mises-Hencky-Huber-Theory）. σp_1 and σp_2 are principal stresses calculated from the shearing stress and the vertical stress. Stress status of the section is almost in pure shearing because absolute value of σp_1 and σp_2 is almost equal to the shearing stress.

Explanation of Fig.-3-18（bending-torsion）: Shear is abbreviation of shearing stress which corresponds to the sum of simple torsion stress, bending-torsion stress and shearing stress relating to bending. Vertical is abbreviation of vertical stress which corresponds the sum of vertical stress by sectional warping and stresses by bending around X and Y axes. Resultant is abbreviation of resultant stress which is calculated according to the shearing strain energy theory（Mises-Hencky-Huber-Theory）. σp_1 and σp_2 are principal stresses calculated from the shearing stress and the vertical stress. Stress status of the section is almost in pure shearing because absolute value of σp_1 and σp_2 is almost equal to the shearing stress. The warping stress which corresponds to majority of the vertical stress may affect sectional stress to certain extent but is not dominant. The maximum shearing stress increase rate of bending-torsion versus simple torsion is about 86 %. Increase of stress occurs between a and b（bottom plate）and c and d（top plate）, and decrease occurs between b and c and d and a（side plates）.

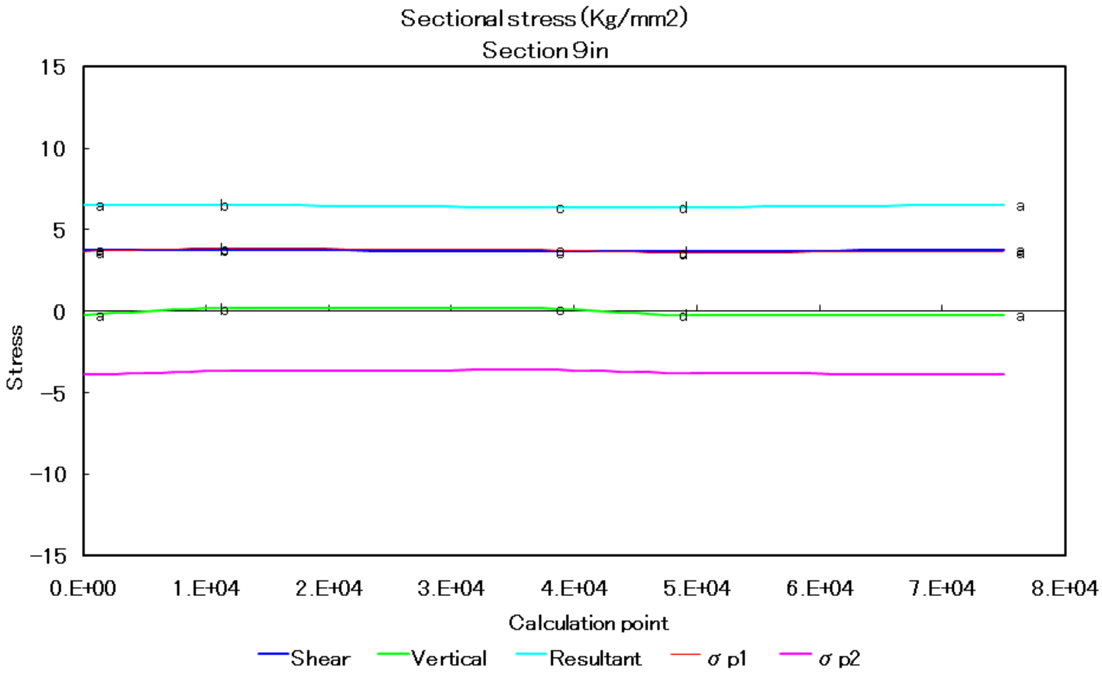


Fig.-3-17 Sectional stress of the simple torsion theory（Rectangular section）

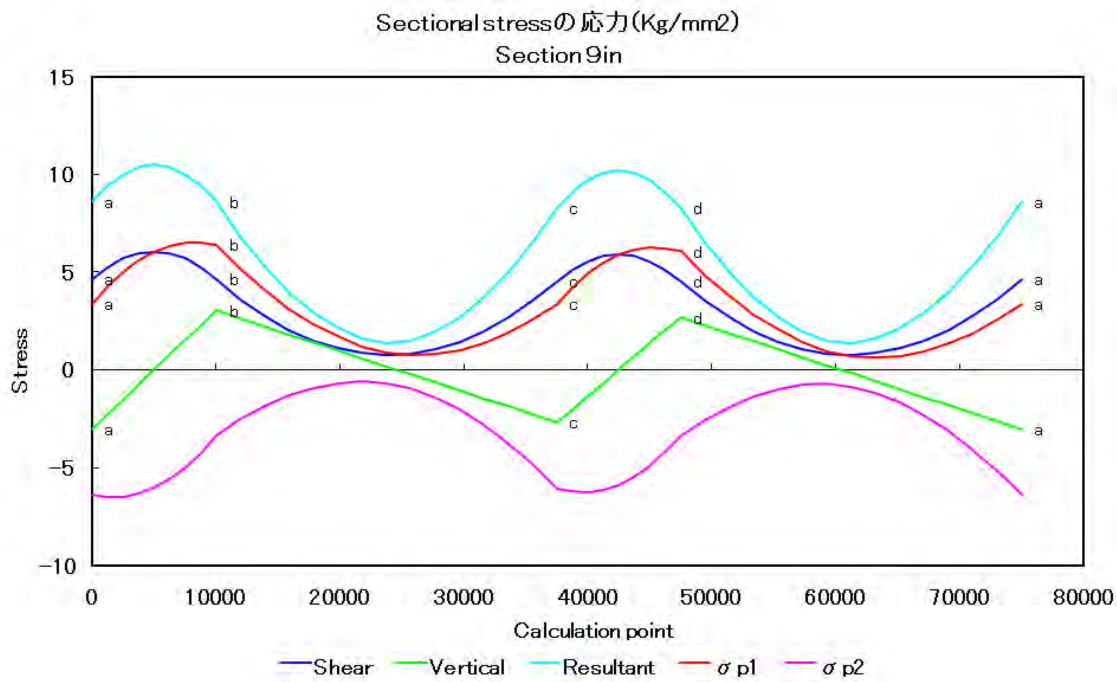


Fig.-3-18 sectional stress of the bending-torsion theory（Rectangular section）

４． Mitigation of warping

It is shown in the previous article that shearing stress on a section meanders with the existence of bending-torsion and its peak value remarkably increases sometimes. A main cause of the meander is the shearing stress which constitutes bending-torsion moment of the section. Since this stress is in equilibrium with vertical stress of sectional warping, it is estimated that the vertical stress as well as the stress meandering will vanish if there is no warping.

Stress is in proportion to the product of a form coefficient, deformation and a spring constant*[1]. If a form coefficient reduces by sectional warping reduction, both shearing and vertical stress of bending-torsion may decrease or vanish (patent pending). On the other hand, the form coefficient reduction will result in section modulus decrease. Because intensity of deformation is equal to internal force divided by the product of a section modulus and a spring constant*[2], the section modulus decrease will bring out deformation increase which may cancel the effect of the form coefficient reduction.

The most important section modulus controlling deformation of a torsion type structure is Ｊｔ*[3]. The product of Ｊｔ and a spring constant is a sectional rigidity which resists simple torsion moment. Although Ｊｔ decreased according to the form coefficient reduction, a gate weight cut is possible by making up the lost Ｊt with help of cross section form change.

The object function of optimum design is construction cost including fabrication. Although the purpose of the form coefficient operation was a gate weight cut, an optimum design of torsion type structure including this operation is possible.

４．１ Condition of zero warping

Fig.-4-1 shows s axis which is set along a center line of thin shell making up a closed section shown in the figure where ｄs is a small quantity at arbitrary point on s, t is

*1Contents of each item are explained at (1) of Appendix 1 Technical key factors.

*2Contents of each item are explained at (1) of Appendix 1 Technical key factors.

*3Relation between θ and Ｊｔ, θ and warping, θ and vertical stress and θ and shearing stress are explained at (1) and (3) of Appendix 1 Technical key factors.

thickness of shell at the point, S is a shear center of the closed section and r_s is distance between the shear center of closed section and a tangent at the arbitrary point. And the area surrounded by s axis is denoted by A_s.

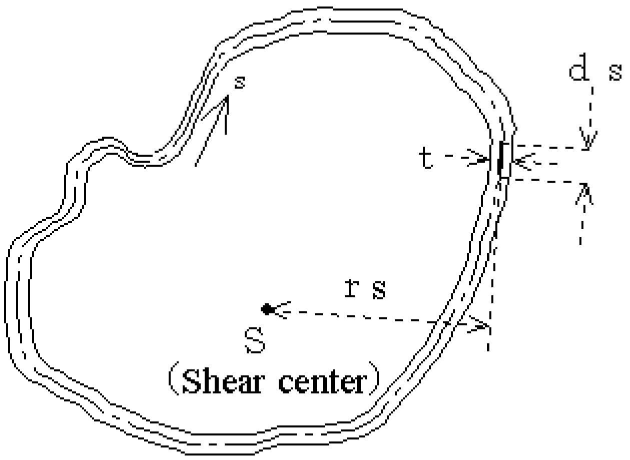


FIg.-4-1 The s axis on a closed section of thin shell

Warping function Ψ of a closed section of thin shell in Fig.-4-1 is given by below formula.

$$\Psi = \Psi_0 - \int_0^s r_s \, ds + 2A_s \int_0^s \frac{1}{t} ds \div \oint \frac{ds}{t}$$

Ψ_0 of above formula is a warping constant which is equal to Ψ at integration start point and is given by below formula.

$$\Psi_0 = \left(\oint t \int_0^s r_s \, ds \, ds - 2A_s \div \oint \frac{ds}{t} \cdot \oint t \int_0^s \frac{1}{t} ds \, ds \right) \div \oint t \, ds$$

Warping function Ψ and a warping constant Ψ_0 are going to be zero, when formula (1) below is satisfied.

$t \times r_s =$ constant on each section $= C$ ・・・・(1)

Sectional warping disappears when Ψ is zero, and vertical stress which is in proportion to Ψ and bending-torsion shearing stress which is in equilibrium with the vertical stress are also zero. In short, **formula (1) is a condition of zero warping**.

4．2 Examples of zero warping

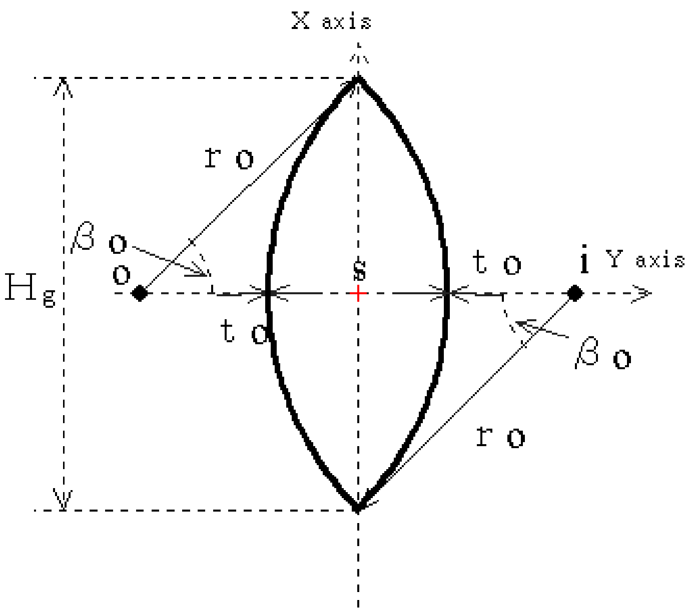
Two examples, a rectangular section and a lens section which is a kind of fish belly section, are presented. Formula (1) is applied to achieve a zero warping status.

4．2．1 A lens section

Fig.-4-2 shows scantlings of a closed lens section of thin shell. As a shearing center coincides with a section center, formula (1) is transformed into formula (2). β is an angle made by r_o and line oi and its range is $0 \leqq \beta \leqq \beta_o$. $\eta(\beta)$ is equal to t/t_o where t is a shell thickness which is a function of β and satisfies the zero warping condition. si is a line si.

$$\eta(\beta) = (r_o - \overline{si}) \div [r_o - \overline{si} \times \cos(\beta)] \quad \cdots\cdots(2)$$

A warping function and a shear flow of bending-torsion for the case of Fig.-4-2 are shown first then a sectional form which satisfies the zero warping condition is shown.



Scantlig of lens section

Item	Data	Unit
Hg=	6000	mm
r o=	6000	mm
β o° =	30	°
t i=	20	mm

Fig.-4-2 Lens section

Fig.-4-3 is a warping function Ψ and a shear flow of bending-torsion for a lens section shown on Fig.-4-2. Ψ is a form coefficient of warping and vertical stress and the shear flow divided by t_0 is a form coefficient for shearing stress of bending-torsion. Distributions of warping and stresses are in proportion to these graphs.

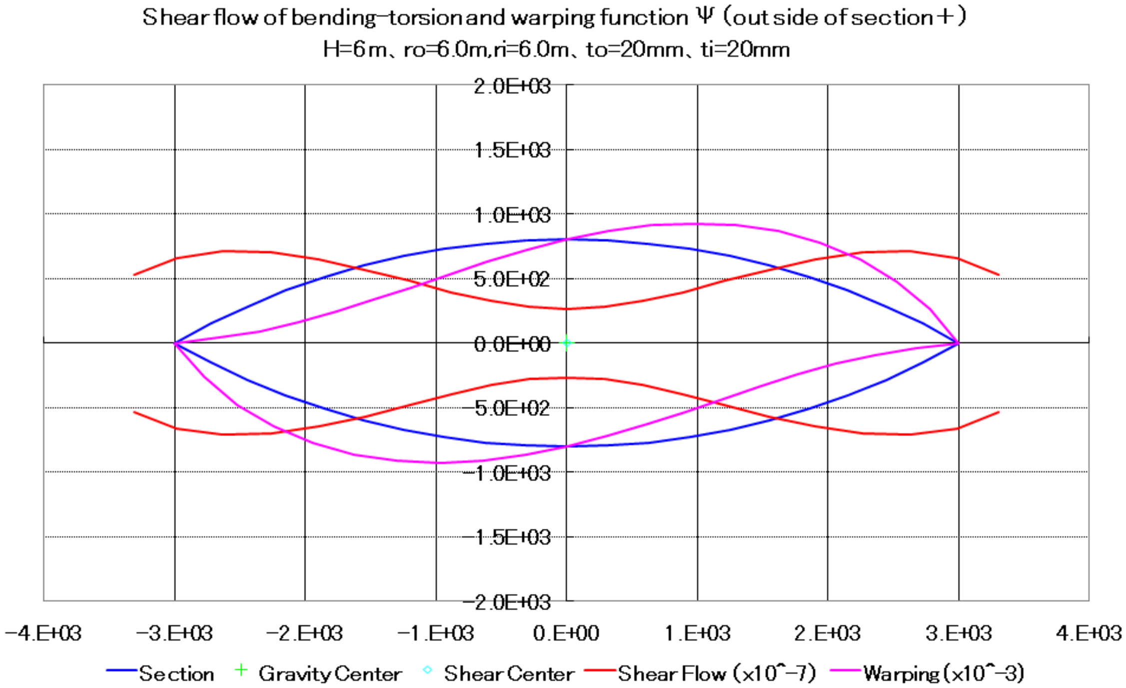
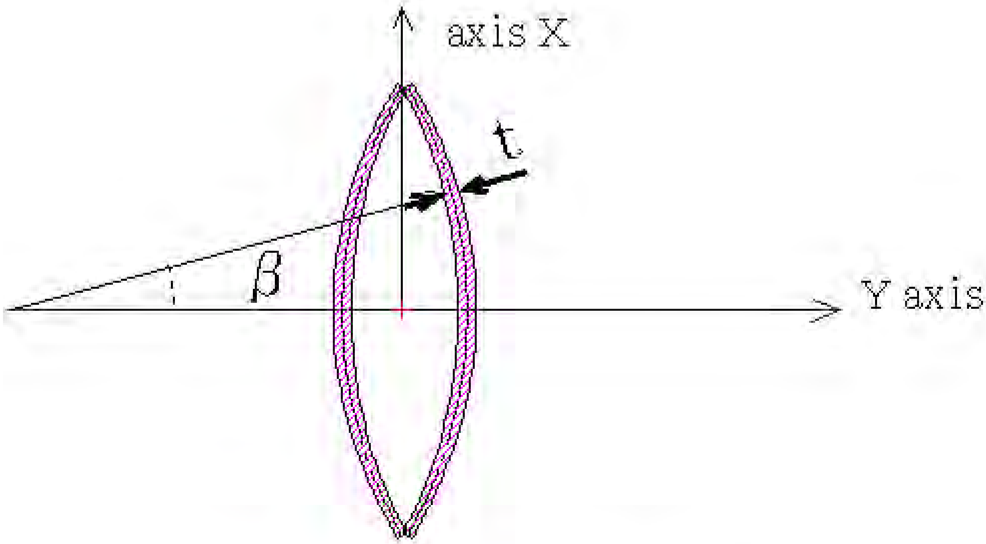


Fig.-4-3 Warping function Ψ and shear flow of bending-torsion (lens section)

Fig.-4-4 shows a form of lens section which satisfies a zero warping condition. Calculated points of formula (2) are joined together by curve lines and the thickness is shown in 10 times scale. The calculation was made on eleven points which distribute uniformly over a quarter of the section. The shell thickness on the Y axis is t_o which is original thickness of Fig.-4-2. The bending-torsion was completely excluded and shear flow and warping shown on Fig.-4-3 vanished.



Zero warping plate thickness
(Lens section)

NO	β °	β rad.	η	t (mm)
0	0	0.000	100.0%	20.0
1	3	0.052	99.1%	19.8
2	6	0.105	96.6%	19.3
3	9	0.157	92.6%	18.5
4	12	0.209	87.6%	17.5
5	15	0.262	81.9%	16.4
6	18	0.314	76.0%	15.2
7	21	0.367	70.0%	14.0
8	24	0.419	64.1%	12.8
9	27	0.471	58.7%	11.7
10	30	0.524	53.6%	10.7

Fig.-4.4 Lens section which satisfies a zero warping condition

４．２．２ A rectangular section

A section of Fundamental case shown on Fig.-3-1 is modified step by step until warping on the section disappears completely. A state of decrease in warping and torsion stress on the section is presented in below two ways.

1) Decrease in warping function and shear flow on a section

2) Change in factors of warping and results of structural analysis

Step by step method of Fundamental section approaching a zero warping condition is explained specifically, before the state of decrease in warping is presented. Zero warping is realized by plate thickness selection as was seen at a lens section. Fundamental section is shown on Fig.-4-5 after setting $t_{f1} = t_{f2} = t_f$ and $t_{w1} = t_{w2} = t_w$ on Fig.-3-1. As a shearing center coincides with an axial center, zero warping condition formula (1) is transformed into formula (3).

$$t_f \times L_w = t_w \times L_f \quad \cdots\cdots (3)$$

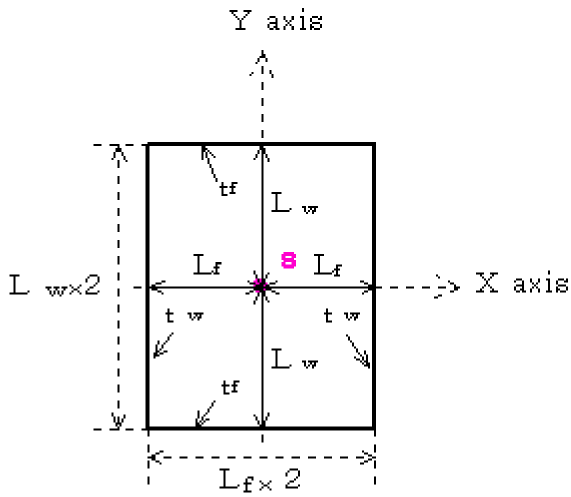


Fig.-4-5 Fundamental section (Axial center = shear center)

There are two choices to realize a zero warping condition, one is to increase t_w, another is to decrease t_f. Suppose purpose of a zero warping condition is gate weight saving, step by step approaching method is to decrease t_f step by step until t_f arrives at $t_w \times L_f \div L_w$ 12.4 mm (a zero warping condition).

(１) Warping function Ψ and shear flow

Fig.-4-6 thru Fig.-4-9 show a state of decrease in warping function and shear flow. Fig.-4-6 is Fundamental section (t f=34 mm). Fig.-4-7 is a section of t f=16 mm, and Fig.-4-8 is t f=14mm. Fig.-4-9 is a zero warping section (t f=12.4 mm).

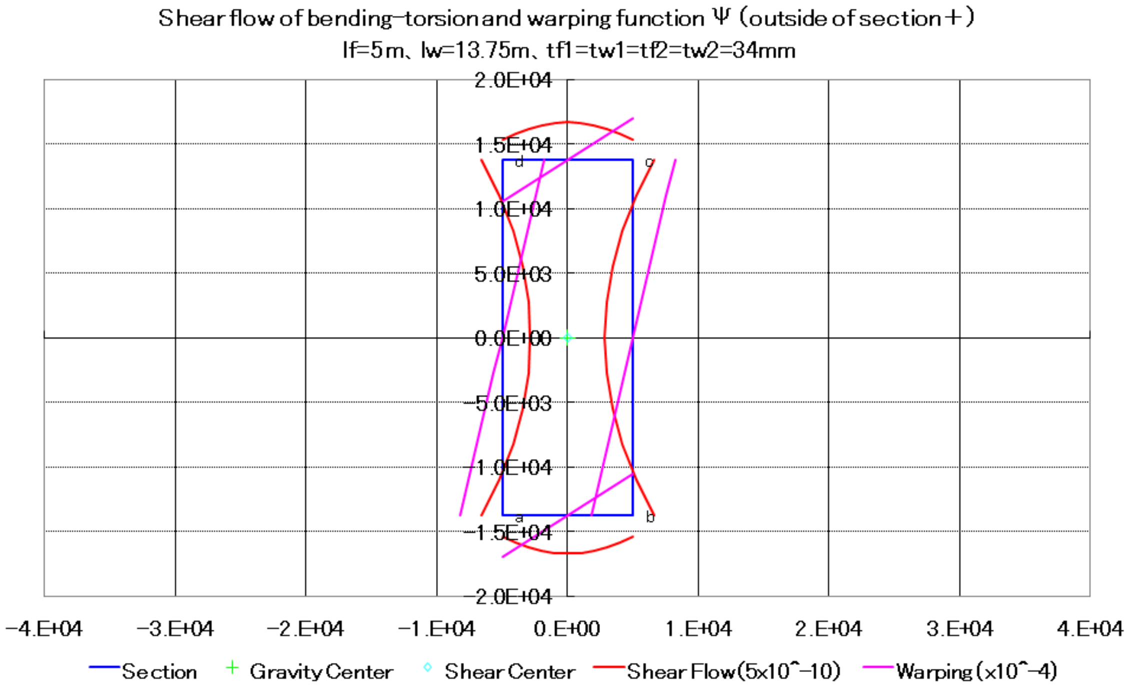


Fig.-4-6 Warping function and shear flow of bending-torsion

(Fundamental section: t f=34 mm)

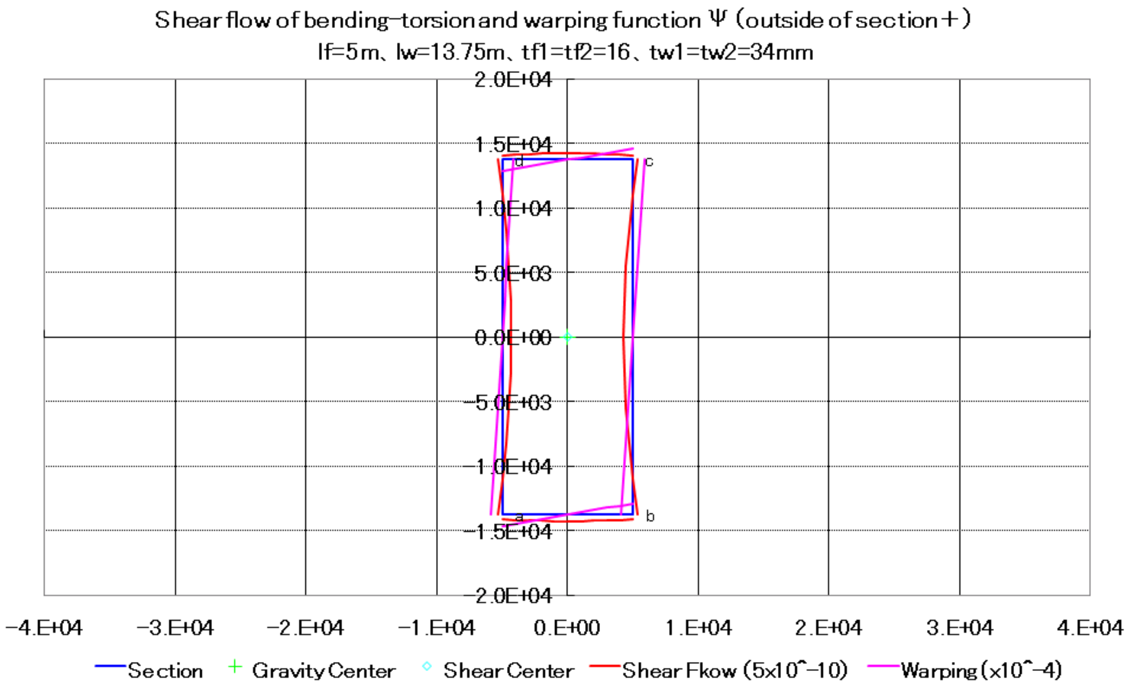


Fig.-4-7 Warping function and shear flow of bending-torsion (t f=16 mm)

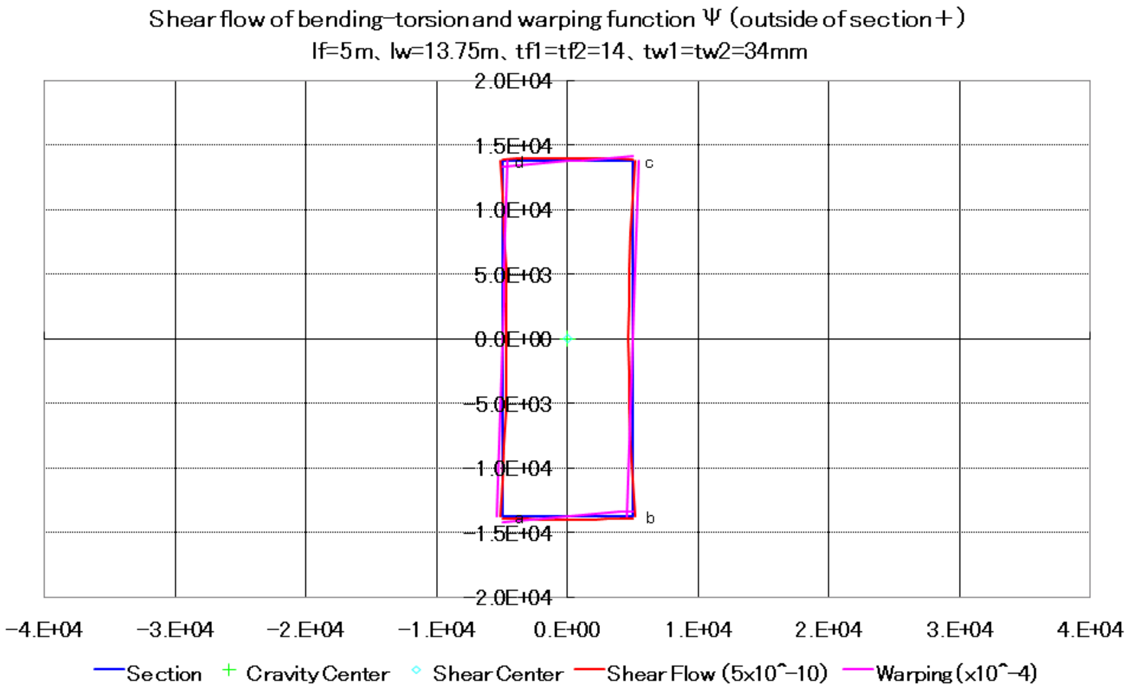


Fig.-4-8 Warping function and shear flow of bending-torsion（ｔｆ=14 mm）

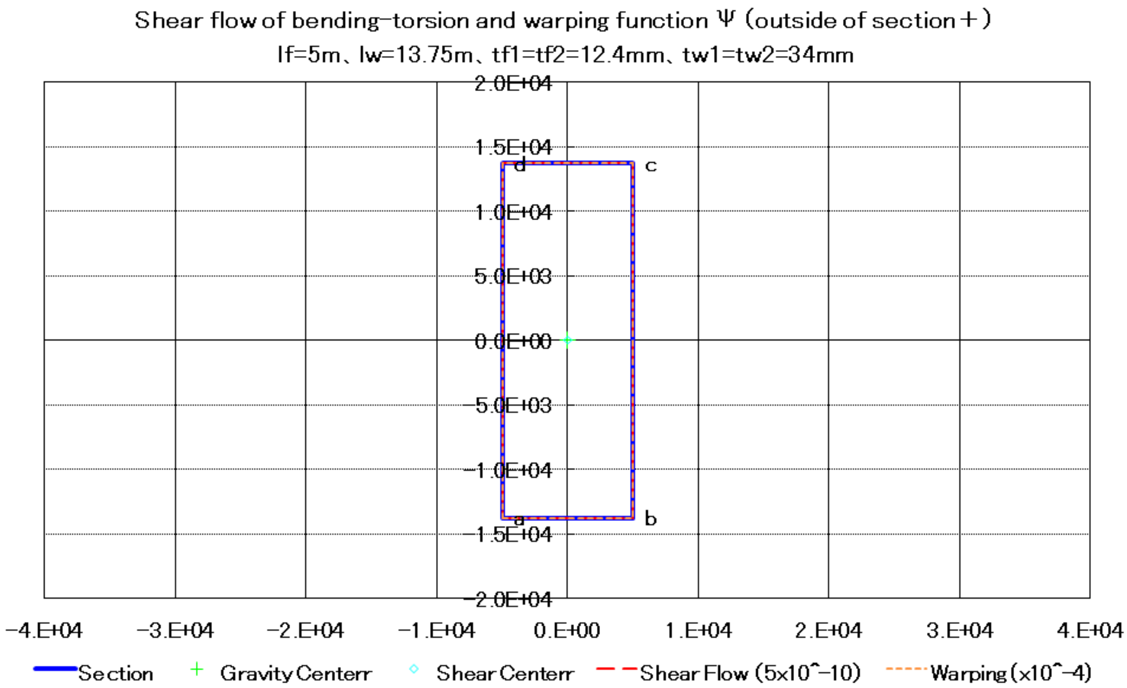


Fig.-4-9 Warping function and shear flow of bending-torsion
（Zero warping section: ｔｆ=12.4 mm）

（２） Factors of warping and structural analyses

Purpose of this item is as follows.
1）To show quantitatively a status of decrease in sectional warping.
2）To show influence on the whole structure of the zero warping operation.
For a reference, the structural analysis was carried out according to the elastic equation method of bibliography（6）.

Fig.-4-10 shows change in factors of warping and influence on structural analysis. The lateral axis is plate thickness t_f, and the vertical axis is factors of warping and results of structural analysis whose indication is in % of Fundamental case.

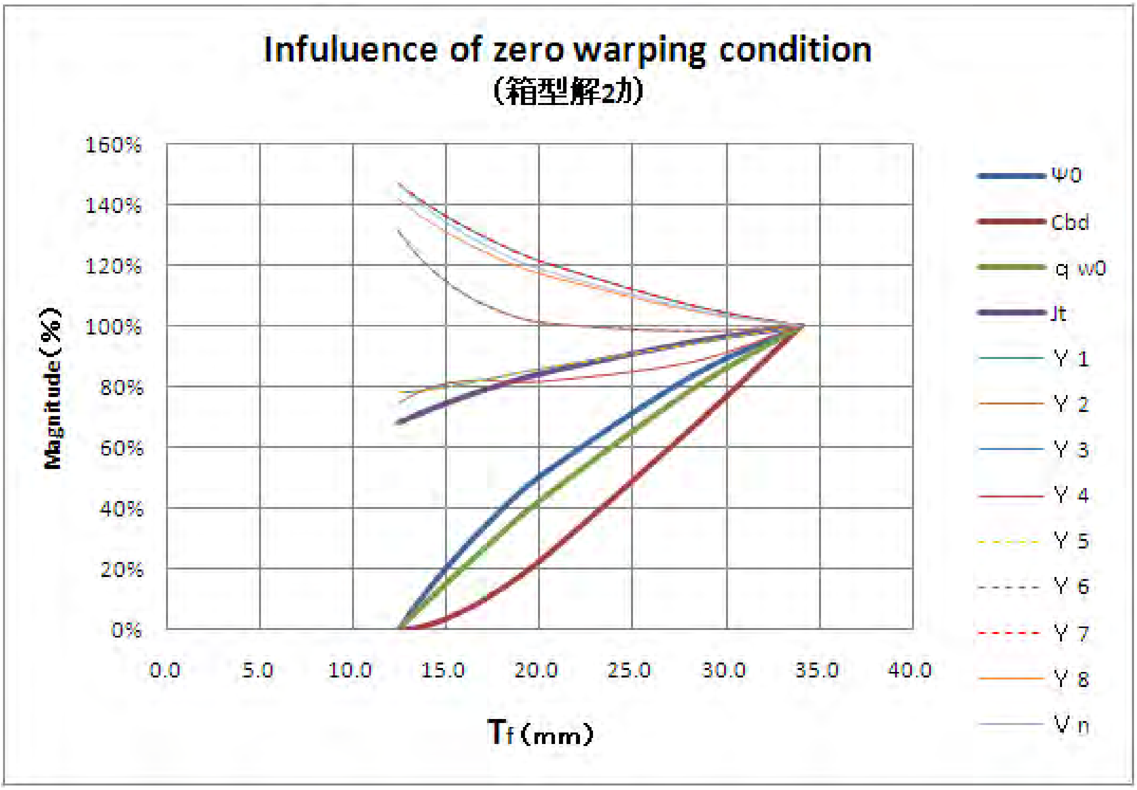


Fig.-4-10 Change in factors of warping and influence on structural analysis

Explanation of factors of warping: A warping constant Ψ_0 which relates to warping magnitude and a shear flow constant q_{w0} (value at integration starting point of shear flow q_w) which relates to shearing stress of bending-torsion are rapidly approaching 0 at t_f=12.4mm. Two section moduli, C_{bd}（bending-torsion）and J_t (simple torsion), are also decreasing. Decrease of J_t is very important because J_t is a leading factor in deformation control of a torsion type structure. Although J_t decrease due to zero warping operation is not unreasonable, there is a possibility of cancelling the form coefficient reduction effort because of an increase in twisting angle θ of the structure.

Explanation of structural analysis: Results of structural analysis shown on the figure are V_n (displacement in Y direction of a shear center at a free end), and Y_1 thru Y_8 (statically indeterminate reaction forces in Y direction of pin supports). As a result of the zero warping operation, following are observed.

1) A remarkable increase of V_n (due to a big increase of θ)

2) Y_3 thru Y_5 of span center portion increased and others of span end portions decreased.

3) Above statement suggests an increase of bending deformation, which is still secondary matter.

4．3 Optimum design

It is shown that the form coefficient reduction will result in section modulus decrease which will bring out stress increase due to increase of twisting angle θ. As the stress increase can be controlled by making up the lost J_t with help of a cross sectional form change, an optimum design including the zero warping condition is possible.

4．3．1 Method of a sectional form change

Although there are two choices, L_f and t_w, as a measure of the form change to increase J_t, the latter does not meet the purpose which is a gate weight cut. Rate of J_t increasing due to L_f increase is much more than rate of the form coefficients (Ψ_0 and q_{w0}) increasing and rate of 2nd and third derivation of θ decreasing due to J_t increase is much more than decreasing rate of first derivation of θ. Accordingly, it is concluded that the L_f increase is an appropriate measure of reducing bending-torsion stress.

A step by step method of t_f approaching the zero warping condition was introduced at 4．2．2. Let t_f at each step be denoted by $t_{f\,step}$. A step by step method is also introduced for the a cross sectional form change and, in order to maintain a distance from a zero warping condition, t_f at each step is rounded out in mm scale after it is calculated by formula (4).

$$t_f = t_{f\,step} \times L_{f\,increased} \div 5000 \text{ (mm)} \qquad \cdots\cdots (4)$$

where, $L_{f\,increased}$: Increased L_f

４．３．２ Method of optimum design

Although purpose of a zero warping condition is gate weight reduction, that of optimum design is cost cut.

As cost composing factors include material, fabrication, transportation, site construction, maintenance, operation etc., minimum gate weight can not necessarily be minimum cost. For instance, there is an option that a high tensile steel plate of specially ordered thickness is fit in the shell of stress increased zone to keep minimum gate weight. But it may be better idea in cost mind to increase gate weight in order to use ordinary strength steel plate of commercial thickness since cost of material and fabrication in the option is more expensive.

So far, stress which yields corresponding to whole structural deformation due to simple torsion, bending-torsion, warping, bending etc. is considered in the study, a torsion type structure which is planned according to a zero warping condition may not be a minimum weight after stability of structural parts or stress which yields corresponding to partial deformation such as bending due to applied hydraulic pressure on gate plates and their stiffeners, bending due to reaction forces of pin supports and support ends etc. are considered.

Suppose an actual conventional measure to find out an optimum design in cost is to select best one among multiple plans, an optimum design should be selected from not only a zero warping point but also a surface made by the two lines one of which is an approaching line to the zero point and other is a cross sectional form change line to make up the lost J_t.

Fig.-4-11 and Fig.-4-12 show image of the optimum design procedure. Fig.-4-11 is a design flow chart and Fig.-4-12 is a selection table of the zero warping condition. A purpose of the selection table is to impose the zero warping condition on the design flow and the table is expected to be a guide of decreased shell thickness setting at S_2. This table set is supposed to be prepared for each design condition and has been corresponding to the example of a rectangular section shown on Fig.-3-14.

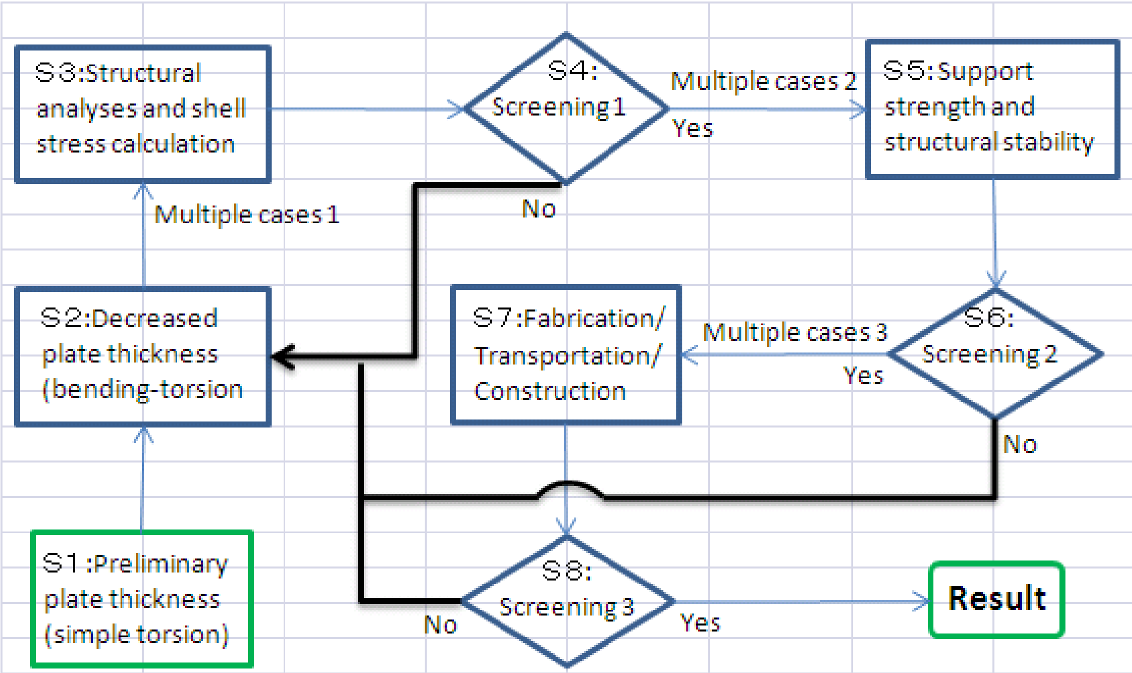


Fig.-4-11 Image of a design flow chart

Preliminary thickness: 34 mm (t_f) (Red: zero warping)

L_f	t_f step (mm)													
(mm)	12.4	13	14	15	16	17	18	20	22	24	26	28	30	34
5000	12.4	13	14	15	16	17	18	20	22	24	26	28	30	34
5100	12.6	14	15	16	17	18	19	21	23	25	27	29	31	35
5200	12.9	14	15	16	17	18	19	21	23	25	28	30	32	36
5300	13.1	14	15	16	17	19	20	22	24	26	28	30	32	37
5400	13.4	15	16	17	18	19	20	22	24	26	29	31	33	37
5500	13.6	15	16	17	18	19	20	22	25	27	29	31	33	38
5600	13.8	15	16	17	18	20	21	23	25	27	30	32	34	39
5900	14.6	16	17	18	19	21	22	24	26	29	31	34	36	41
6700	16.6	18	19	21	22	23	25	27	30	33	35	38	41	46
7500	18.5	20	21	23	24	26	27	30	33	36	39	42	45	51

Preliminary thickness: 34mm (Weight %) (Red: zero warping)

L_f	t_f step (mm)													
(mm)	12.4	13	14	15	16	17	18	20	22	24	26	28	30	34
5000	83	84	84	85	86	87	87	89	91	92	94	95	97	100
5100	83	85	85	86	87	88	89	90	92	93	95	97	98	101
5200	84	85	86	86	87	88	89	90	92	94	96	98	99	103
5300	84	85	86	87	87	89	90	92	93	95	97	98	100	104
5400	85	86	87	88	89	89	90	92	94	95	98	100	101	105
5500	85	86	87	88	89	90	91	92	95	97	98	100	102	106
5600	85	87	87	88	89	91	92	94	95	97	100	101	103	108
5900	87	88	89	90	91	93	94	96	97	100	102	105	107	111
6700	91	92	93	95	96	98	100	102	105	108	110	113	116	122
7500	95	97	98	100	102	104	105	109	112	116	119	123	126	133

Preliminary thickness: 16 mm (t_f) (Red: zero warping)

L_f	t_f step (mm)													
(mm)	5.8			6	7	8	9	10	11	12	13	14	15	16
5000						8	9	10	11	12	13	14	15	16
5100						9	10	11	12	13	14	15	16	17
5200						9	10	11	12	13	14	15	16	17
5300					8	9	10	11	12	13	14	15	16	17
5400					8	9	10	11	12	13	15	16	17	18
5500					8	9	10	11	13	14	15	16	17	18
5600					8	9	11	12	13	14	15	16	17	18
5900				8	9	10	11	12	13	15	16	17	18	19
6700	7.8			9	10	11	13	14	15	17	18	19	21	22
7500	8.7			9	11	12	14	15	17	18	20	21	23	24

Preliminary thickness: 16mm (Weight %) (Red: zero warping)

L_f	t_f step (mm)													
(mm)	5.8			6	7	8	9	10	11	12	13	14	15	16
5000						87	88	90	92	93	95	97	98	100
5100						89	90	92	94	95	97	99	101	102
5200						89	91	92	94	96	98	99	101	103
5300					87	89	91	93	95	96	98	100	102	103
5400					88	90	91	93	95	97	100	102	104	106
5500					88	90	92	94	97	99	101	103	105	106
5600					88	90	94	96	98	99	101	103	105	107
5900				89	91	93	95	97	99	103	105	107	109	111
6700				93	96	98	102	105	107	111	114	116	120	122
7500	95			96	101	103	108	111	116	118	123	126	131	133

Preliminary thickness: 14 mm (t_f) (Red: zero warping)

L_f	t_f step (mm)													
(mm)	5.1				5	6	7	8	9	10	11	12	13	14
5000								8	9	10	11	12	13	14
5100							8	9	10	11	12	13	14	15
5200							8	9	10	11	12	13	14	15
5300							8	9	10	11	12	13	14	15
5400							8	9	10	11	12	13	15	16
5500							8	9	10	11	13	14	15	16
5600							8	9	11	12	13	14	15	16
5900						8	9	10	11	12	13	15	16	17
6700						9	10	11	13	14	15	17	18	19
7500	7.6				8	9	11	12	14	15	17	18	20	21

Preliminary thickness: 14mm (Weight %) (Red: zero warping)

L_f	t_f step (mm)													
(mm)	5.1				5	6	7	8	9	10	11	12	13	14
5000								89	90	92	94	96	98	100
5100							89	91	93	95	97	99	101	102
5200							89	91	93	95	97	99	101	103
5300							89	92	94	96	98	100	102	104
5400							90	92	94	96	98	100	104	106
5500							90	92	94	96	101	103	105	107
5600							90	93	97	99	101	103	105	107
5900						91	94	96	98	100	103	107	109	112
6700						96	99	101	107	109	112	117	119	122
7500					96	99	105	108	113	116	122	125	130	133

Preliminary thickness: 12 mm (t_f) (Red: zero warping)

L_f	t_f step (mm)													
(mm)	4.4						5	6	7	8	9	10	11	12
5000										8	9	10	11	12
5100									8	9	10	11	12	13
5200									8	9	10	11	12	13
5300									8	9	10	11	12	13
5400									8	9	10	11	12	13
5500									8	9	10	11	13	14
5600									8	9	11	12	13	14
5900								8	9	10	11	12	13	15
6700								9	10	11	13	14	15	17
7500	6.5						8	9	11	12	14	15	17	18

Preliminary thickness: 12mm (Weight %) (Red: zero warping)

L_f	t_f step (mm)													
(mm)	4.4						5	6	7	8	9	10	11	12
5000										91	93	96	98	100
5100									91	94	96	98	101	103
5200									92	94	96	99	101	103
5300									92	95	97	99	102	104
5400									93	95	97	100	102	105
5500									93	95	98	100	105	108
5600									93	96	101	103	106	108
5900								94	97	100	102	105	107	113
6700								100	103	106	112	115	118	124
7500							100	103	110	113	120	123	130	133

Fig.-4-12 Image of a selection table of the zero warping conditions

５． Conclusion

(1) A torsion type closed thin shell section is overwhelmingly superior to a bending type structure.

(2) There are two types of structural torsion, the simple torsion and the bending-torsion.

(3) Sectional shearing stress distribution of simple torsion moment is almost uniform.

(4) The sectional shearing stress distribution meanders by additional bending-torsion moment.

(5) In some case, peak stress of additional bending-torsion is much more than 200% of simple torsion alone.

(6) Stress is in proportion to a form coefficient × deformation × a spring constant.

(7) Form coefficient can be diminished by eliminating sectional warping.

(8) Ｊｔ will reduce due to warping reduction operation, and stress and deformation increase accordingly.

(9) Control of stress increase is possible by making up the lost Ｊｔ with help of a cross sectional form change.

(10) An optimum design including the zero warping condition is possible.

Appendix 1. **Technical key factors of a torsion type structure**

(1) Factors of stress and deformation (their contents and relations)

1) Stress = Form coefficient × Deformation × Spring constant

2) Deformation = Internal force ÷ Sectional rigidity

3) Sectional rigidity = Section modulus × Spring constant

Table－1 Factors of stress (contents)

Stress	Form coefficient	Deformation	Spring constant
σb (bending)	x 、 y	$\ddot{x}$ 、 $\ddot{y}$	E
τb (bending)	$\frac{q_{bx}}{t}$ 、 $\frac{q_{by}}{t}$	$\dddot{x}$ 、 $\dddot{y}$	E
τs (simple torsion)	$\frac{q_{s}}{t}$	$\dot{\theta}$	G
σz (bending-torsion)	Ψ	$\ddot{\theta}$	E
τw (bending-torsion)	$\frac{q_{\omega}}{t}$	$\dddot{\theta}$	E

Table－2 Factors of deformation (contents)

Deformation	Internal force	Sectional rigidity	
		Section modulus	Spring constant
$\ddot{x}$ 、 $\ddot{y}$	m_{by} 、 m_{bx}	I_y 、 I_x	E
$\dddot{x}$ 、 $\dddot{y}$	Q_x 、 Q_y	I_y 、 I_x	E
$\dot{\theta}$	T_s	J_t	G
$\ddot{\theta}$	(?)	$(J_t \times C_{bd})^{0.5}$	$(G \times E)^{0.5}$
$\dddot{\theta}$	T_w	C_{bd}	E

（２）Relations of stress to ｔ and Ｓ（degree of effect）.

Red colored factors in Table－１ and Table－２ are affected from ｔ and Ｓ. Table－３ shows degree of ｔ and Ｓ effect on section moduli. Table－４ shows degree of ｔ and Ｓ effect on stress. Effect degree of deformation is invert of section modulus and effect degree of stress is a product of form coefficient and deformation. Effect degree analysis is based upon geometric similarity of sectional form and of t distribution.

Table－３ Degree of ｔ and Ｓ effect on section modulus

Section modulus	Effect degree of t and S
I_y 、 I_x	$t^1 \cdot s^3$
J_t	$t^1 \cdot s^3$
$(J_t \times C_{bd})^{0.5}$	$t^1 \cdot s^4$
C_{bd}	$t^1 \cdot s^5$

Table－４ Degree of ｔ and Ｓ effect on stress

Stress	Effect degree of t and S		
	Form coefficient	Deformation	Stress
σb (bending)	S	$t^{-1} \cdot s^{-3}$	$t^{-1} \cdot s^{-2}$
τb (bending)	S^2	$t^{-1} \cdot s^{-3}$	$t^{-1} \cdot s^{-1}$
τs (simple torsion)	S	$t^{-1} \cdot s^{-3}$	$t^{-1} \cdot s^{-2}$
σz (bending-torsion)	S^2	$t^{-1} \cdot s^{-4}$	$t^{-1} \cdot s^{-2}$
τw (bending-torsion)	S^2	$t^{-1} \cdot s^{-5}$	$t^{-1} \cdot s^{-3}$

（３）Factors of deformation（their contents and relations）

1）Displacement＝Deformation×Form coefficient

Table－５ Factors of displacement（contents）

Displacement		Defor-mation	Form coefficient
Direction	Mark		
X direction	x	x	1
Ydirection	y	y	1
X direction	u	θ	$-(y-L_{py})$
Ydirection	v	θ	$x-L_{px}$
Warping (Z)	w	$\dot{\theta}$	Ψ

Appendix ２．Bibliography

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