Gates of Closed Thin Shell Section

Why is Torsion Type Superior to Bending Type?

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Contents

１.　Introduction

Characteristics of torsion type structure are compared to bending type structure to make its superiority being clear. The comparison will be made on a closed thin shell section gate as an example.

A torsion type closed thin shell section is overwhelmingly superior in structural functions to a bending type structure because the torsion type resists against external load with second power of the closed area whereas the bending type resists with moment of inertia of thin shell, and this superiority gets more remarkable as the structural span becomes longer.

The superiority of torsion type structure is applicable to comparison of another type of structure such as girder type, axial force type so on.

2． Study on structural key factors

2． 1 Structural key factors

Displacement, deformation, sectional internal force and stress, which are key factors of elastic structural behavior, have relations given by formula (1) thru (3).

Stress ＝ Form coefficient × Deformation × Spring constant ･･････(1)

Deformation ＝ Internal force ÷ Sectional rigidity
＝ Internal force ÷ (Section modulus × Spring constant) ･･････(2)

Displacement ＝ Deformation × Form coefficient ･･････(3)

The form coefficients of the formula (1) and (3) are relating to sectional shapes of structure but their contents are different.

2． 2 Factors of stress

Table－2－1 shows contents of stress factors which are components of formula (1).

Table－2－1 Factors of stress (contents)

Stress	Form coefficient	Deformation	Spring constant
σb (bending)	x 、 y	$\ddot{x}$ 、 $\ddot{y}$	E
τb (bending)	$\frac{q_{wx}}{t}$ 、 $\frac{q_{wy}}{t}$	$\dddot{x}$ 、 $\dddot{y}$	E
τs (simple torsion)	$\frac{q_s}{t}$	$\dot{\theta}$	G
σz (bending-torsion)	Ψ	$\ddot{\theta}$	E
τw (bending-torsion)	$\frac{q_\omega}{t}$	$\dddot{\theta}$	E

2． 3 Factors of deformation

Table－2－2 shows contents of deformation factors which are components of formula (2).

Table－２－２　Factors of deformation（contents）

Deformation	Internal force	Sectional rigidity	
		Section modulus	Spring constant
$\ddot{x}$ 、 $\ddot{y}$	m_{by} 、 m_{bx}	I_y 、 I_x	E
$\dddot{x}$ 、 $\dddot{y}$	Q_x 、 Q_y	I_y 、 I_x	E
$\dot{\theta}$	T_s	J_t	G
$\ddot{\theta}$	(?)	$(J_t \times C_{bd})^{0.5}$	$(G \times E)^{0.5}$
$\dddot{\theta}$	T_w	C_{bd}	E

２． ４　Factors of displacement

Table－２－３ shows contents of displacement factors which are components of formula（3）.

Table－２－３　Factors of displacement（contents）

Displacement		Defor-mation	Form coefficient
Direction	Mark		
X direction	x	x	1
Ydirection	y	y	1
X direction	u	θ	$-(y-L_{py})$
Ydirection	v	θ	$x-L_{px}$
Warping (Z)	w	$\dot{\theta}$	Ψ

３.　Bending type structure and torsion type structure

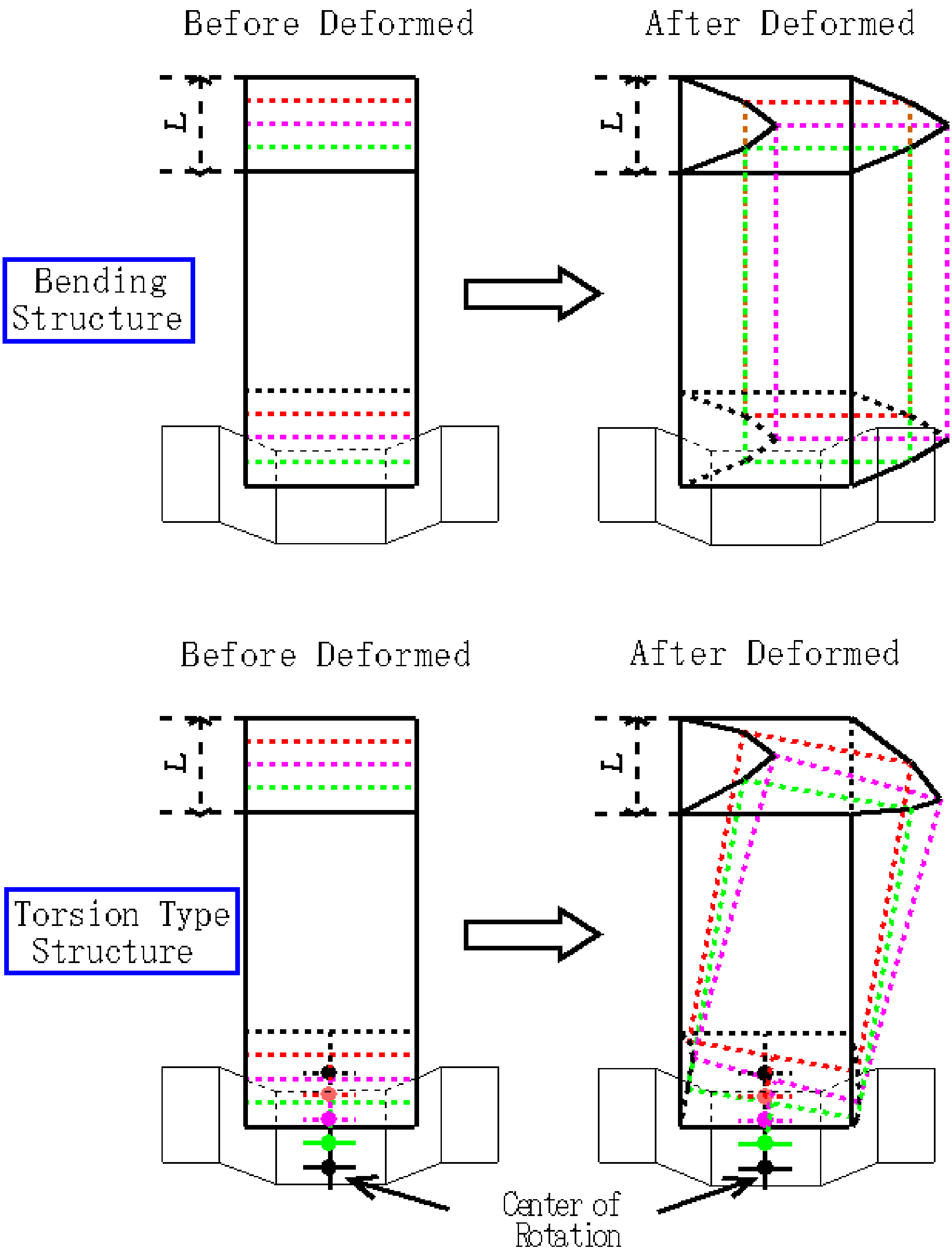

Fig.－３－１　Characteristic Deformation of Bending and Torsion Type Structure

Fig.－３－１ shows characteristic deformations of the torsion and bending type structure. L in the figure is clear span of the gates. Suppose deformation of the bending type structure is characterized by parallel movement of the structural section, that of the torsion type structure can be characterized by rotation of the structural section in its plane. The rotation center of the section is the restriction point on the section of the parallel movement. In short, a structure can be either torsion type or bending type according to existence of the restriction point or not. But the structural character of the two types are remarkably different when sectional shape of the structures are closed like a box beam. The bending structure resists loads on the span with help of the sectional moment of inertia whereas the torsion type structure resists with help of a second power of the closed sectional area. The load transmittal routes are also different. The load on the bending structure is transmitted to the supported span ends with the sectional shearing rigidity, whereas the load applied to a section of torsion structure is transmitted to the restriction point on the section and the torsion moment composed of the load and the restriction force of the section is transmitted to the supported span ends with the sectional torsion rigidity. The bending type is 3 dimensional structure wheraas the torsion type may be classified into 2.5 dimensional structure. The torsion type structure has various merits due to structural characteristics different from the bending type and the merits become more remarkable as the structural span becomes longer.

Table－３－１ Leading Section Modulus of Bending Type and Torsion Type Structures

Stress/ Displacement		Lead role in structural behavior		Deformation	Section modulus
		Bending	Torsion		
Stress	σb (Bending)	○	—	$\ddot{x}$ 、 $\ddot{y}$	Iy 、 Ix
	τb (Bending)	—	—	$\dddot{x}$ 、 $\dddot{y}$	Iy 、 Ix
	τs (Simple t.)	—	○	$\dot{\theta}$	Jt
	σz (Bending t.)	—	—	$\ddot{\theta}$	$(Jt \times Cbd)^{0.5}$
	τw (Bending t.)	—	—	$\dddot{\theta}$	Cbd
Displacement	x、y (Bending)	○	—	x、y	Iy 、 Ix
	u、v (Torsion)	—	○	θ	Jt(Remark1)
	w (Warping)	—	—	$\dot{\theta}$	Jt

Remark1：1) This is a section modulus of simple torsion.
2) θ at free turminals of simple and bending torsion are equal.
3) Section modulus of bending torsion is Jt×(Jt÷Cbd) 0.5.

Table－３－１ shows leading section moduli of bending type structure and torsion type structure. The ○ marks show lead roles among stresses and among displacement both of which are connected to section moduli through deformations according to the results of study carried out at preceding article. In short, lead roles of bending type structure are I_x and I_y, and that of torsion type structure is J_t.

4． Superiority of torsion type of structure

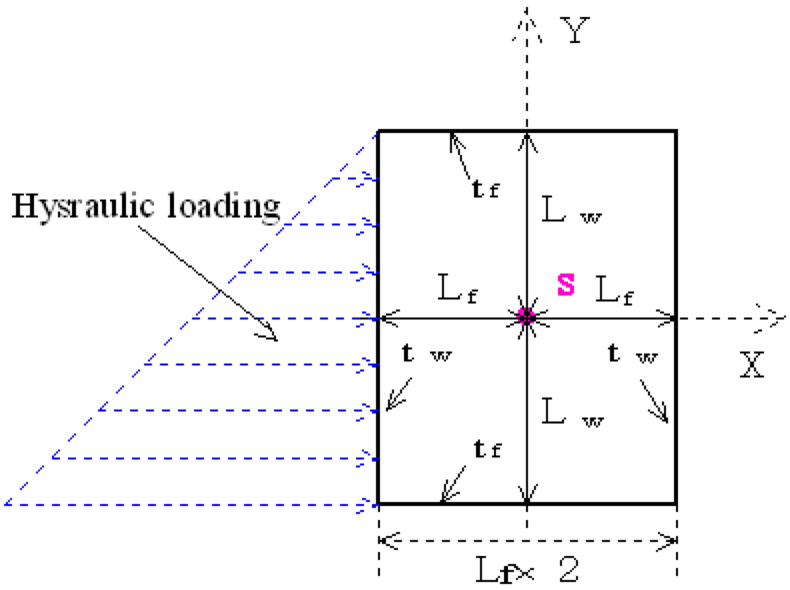

Fig.－4－1 Box type closed thin shell section

A study is made on the box type closed thin shell section bearing triangular hydraulic pressure as show on Fig. －4－1. Sectional internal forces and section moduli to resist the forces of the structures shown on Fig. －3－1 are going to be mathematized. A leading section modulus of the bending type structure is sectional moment of inertia and corresponding internal force is bending moment and a leading section modulus of the torsion type structure is sectional torsion modulus and corresponding internal force is torsion moment.

【 **Bending Type Structure** 】

Section modulus ＝ I_y

$$= 4L_w \times t_w \times L_f^{\,2} + 4t_f \times L_f^{\,3} \div 3 = a_w \times L_f^{\,2} + a_f \times L_f^{\,2} \div 3 \cdots\cdots (4)$$

where a_w and a_f are sectional areas of the thin shells which respectively correspond to webs or flanges of the box type section.

Internal force $= M_x = 4L_w^{\,2} \div 2 \times L^2 \div 8 = (L_w^{\,2} \times L)\, L \div 4 \quad \cdots\cdots (5)$

【Torsion type structure】

Section modulus $=J_t = 4A^2 \div (4L_f/t_f + 4L_w/t_w) = A^2 \div (L_f/t_f + L_w/t_w)$ ‥(6)

where A is a sectional area of closed thin shell section.

Internal force $= 8L_w^3 \div 6 \times L \div 2 = (L_w^2 \times L)\ 2\ L_w \div 3$ ‥‥‥(7)

Formula (4) indicates that all of a_w and one third of a_f in the bending type structure are effective element of the sectional moment of inertia I_y whereas formula (6) indicates that a square of sectional area surrounded by thin shell in the torsion type structure is effective element of torsion section modulus J_t.

Formula (5) gives bending moment in the bending type structure while formula (7) gives torsion moment in the torsion type structure. An essential difference of the two formulas is structural span L of the bending type structure and sectional height $2L_w$ of the torsion type structure that suggests a superiority of the torsion type becomes more remarkable as the structural span becomes longer.

５. Conclusion

Following is conclusion of the study based on a closed thin shell section.

1) The bending type structure resists loads with help of the sectional moment of inertia.

2) The torsion type structure resists loads with help of a square of the area of closed thin shell section.

3) A superiority of the torsion type becomes more remarkable as the structural span becomes longer.

4) But above described superiority of the torsion type is applicable to another type of structures such as sector type, etc.